

Beyond the Standard Model

A. Pomarol

Universitat Autònoma de Barcelona, Bellaterra, Spain

Abstract

In these lectures we briefly cover some of the main lines of research in particle physics beyond the Standard Model

1 Introduction

The Standard Model (SM) of elementary particles together with Einstein's theory of General Relativity for gravity provide us with a remarkable, simple framework to explain, at present, almost all physical processes observed in Nature. Only physics at shorter distances than the Planck length, where gravity must be fully quantized, and some experimental evidence (neutrino masses, dark matter) seem to escape from this general framework and require us to go beyond it. This new physics, however, could appear only at around Planckian energies, thus not providing strong motivations for new feasible experiments testing physics at smaller energies such as the LHC.

A different stimulus for physics beyond the SM, that has inspired a wide range of experiments, has originated from trying to improve our theoretical understanding of the SM. Indeed, certain couplings and masses in the SM, determined by experiments, seem to demand a better explanation, and this has required us to postulate new physics beyond the SM. The purpose of these lectures is to give a brief description of these theories, giving their motivation and predictions at present and future experiments. We have divided the lectures into the following topics:

- The SM of particle physics: symmetries, consistency, and reasons for improvement.
- Grand Unified Theories.
- The strong CP-problem and axions.
- The hierarchy problem.
- Supersymmetry.
- Higgsless models and composite Higgs.
- Extra dimensions.

2 The SM of particle physics: symmetries, consistency, and reasons for improvement

The SM is a quantum field theory whose Lagrangian, that gives us the particle spectrum and interactions, is fixed by local symmetries and the matter content. The local symmetries of the SM are those associated to

1. The Poincaré group.
2. The gauge group $SU(3)_c \times SU(2)_L \times U(1)_Y$.

The matter content, defined by the quantum numbers under the above groups, consists of a fermionic and a scalar sector. The fermionic sector is composed of three copies of fields charged under the gauge

group¹:

	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$
Q_L	3	2	1/3
u_R	3	1	4/3
d_R	3	1	-2/3
l_L	1	2	-1
e_R	1	1	-2

(1)

Each copy corresponds to, what is usually called, a family of particles. They are fields of spin 1/2. The scalar sector contains the Higgs. This is a scalar with charges:

	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$
H	1	2	1

(2)

Invariance under the gauge symmetries requires the presence of additional gauge boson fields, the gluons G_μ , the W_μ and B_μ . These are fields of spin 1. Finally, invariance under the Poincaré group requires the presence of the gravitational field $g_{\mu\nu}$ of spin 2. All these fields mediate interactions between matter fields.

Once the local symmetries and the matter content of the SM are fixed, the Lagrangian is fully determined. Neglecting for the moment gravity whose strength is very small compared to other interactions in particle physics experiments, we have that the renormalizable SM Lagrangian is given by

$$\begin{aligned}
\mathcal{L}_{\text{SM}} = & -\frac{1}{4g'^2} B^{\mu\nu} B_{\mu\nu} - \frac{1}{4g^2} W_a^{\mu\nu} W_{\mu\nu}^a - \frac{1}{4g_s^2} G_a^{\mu\nu} G_{\mu\nu}^a \\
& + i\bar{Q}_L^i \not{D} Q_L^i + i\bar{u}_R^i \not{D} u_R^i + i\bar{d}_R^i \not{D} d_R^i + i\bar{e}_R^i \not{D} e_R^i + i\bar{l}_L^i \not{D} l_L^i \\
& + Y_u^{ij} \bar{Q}_L^i \tilde{H} u_R^j + Y_d^{ij} \bar{Q}_L^i H d_R^j + Y_e^{il} \bar{l}_L^i H e_R^j + h.c. \\
& + |D_\mu H|^2 + \mu^2 |H|^2 - \lambda |H|^4,
\end{aligned}
\tag{3}$$

where $i, j = 1, 2, 3$ label the families. The explanation for each term of this Lagrangian is given in the lectures of A. Pich in this report. Apart from kinetic terms and mass terms, this Lagrangian gives us the interactions between the SM particles. We have gauge interactions whose strengths are measured by g_s , g and g' , Yukawa interactions $Y_{u,d,e}$, and Higgs self-interactions measured by λ . The important feature of the SM is that all these couplings are dimensionless (in natural units $\hbar = c = 1$) and therefore the theory can be extrapolated into a wide range of energies. We will discuss this in more detail later on.

By rotating the three families of fermion fields, we can go to the basis in which the Yukawa couplings can be written as

$$Y_d = V_{\text{CKM}}^\dagger M_d^{\text{diag}} / \langle H \rangle, \quad Y_u = M_u^{\text{diag}} / \langle H \rangle, \quad Y_e = M_e^{\text{diag}} / \langle H \rangle, \tag{4}$$

where $M_{d,u,e}^{\text{diag}}$ are the diagonal fermion mass matrices, V_{CKM} is the Cabibbo-Kobayashi-Maskawa matrix, and $\langle H \rangle$ is the Higgs vacuum expectation value (VEV).

We have measured all parameters of the SM [1], except the Higgs self-coupling λ that determines the Higgs mass $M_h = \sqrt{2\lambda}v$ where $v = \sqrt{2} \langle H \rangle \simeq 246$ GeV. Direct Higgs searches at LEP and the Tevatron, and electroweak precision tests (EWPT) give important constraints on the Higgs mass. One gets $114.4 \text{ GeV} < M_h < 155 \text{ GeV}$ at 95% CL [2].

2.1 Accidental symmetries

The Lagrangian Eq. (3) has extra symmetries that appear 'accidentally' since we did not impose them. These symmetries allow one to explain certain properties of the SM. The most important one is baryon

¹We are using the convention of charges such that the EM charge is defined as $Q = Y/2 + T_3$ where T_3 is the 3rd component of the $SU(2)_L$ generator and Y is the charge under the $U(1)_Y$ group, the so-called hypercharge.

number. This is a global $U(1)_B$ symmetry under which the SM fermions ψ transform as

$$\psi \rightarrow e^{iB\theta}\psi, \quad (5)$$

where θ is the parameter of the transformation and B is the baryon number. We have $B = 1/3$ for the quark fields Q_L , u_R and d_R , while $B = 0$ for leptons. It is not hard to see that, indeed, this is a global symmetry of Eq. (3). This accidental symmetry has an important implication. At the perturbative level², it guarantees that the proton, that is made of three quarks and then has baryon number $B = 1$, cannot decay into lighter leptonic states. This prediction of the SM is supported by the experimental data that, up to now, has not shown any evidence for proton decay [1].

Other accidental symmetries of the SM Lagrangian are the three leptonic global symmetries. These are (1) the electronic lepton symmetry, whose charges are $L_e = 1$ for electrons and the neutrino ν_e and zero for the rest; (2) the muonic lepton symmetry, whose charges are $L_\mu = 1$ for muons and the neutrino ν_μ and zero for the rest; and (3) tauonic lepton symmetry, whose charges are $L_\tau = 1$ for taus and the neutrino ν_τ and zero for the rest. These symmetries forbid leptons to decay into a lighter one plus a photon, e.g., $\mu \rightarrow e\gamma$. Again, this prediction is, at present, supported by experimental data that has shown no evidence for these decays [1]. It also predicts that neutrinos cannot have mass, in clear contradiction with experimental evidences of neutrino oscillations. We will comment on this point later.

The SM Lagrangian also has *approximate* accidental symmetries that play an important role in the understanding of some physical properties of the model. These are global symmetries that are only broken by small couplings in the SM. An important one is the so-called ‘custodial’ $SU(2)_c$ symmetry [3]: in the limit $Y_{u,d,e} = 0$ and $g' = 0$, the SM Lagrangian has an extra global $SU(2)$ symmetry under which the Higgs field H transforms³ as a **2**. The Higgs VEV breaks this symmetry down to the custodial symmetry, $SU(2) \times SU(2)_L \rightarrow SU(2)_c$, under which the physical Higgs h transform as a singlet and the massive W_μ^\pm and Z_μ form a triplet. This symmetry predicts $M_W = M_Z$. When g' is turned on, this prediction is altered to be

$$\frac{M_W^2}{M_Z^2 \cos^2 \theta_W} \equiv \rho = 1, \quad (6)$$

where θ_W is the weak angle. Yukawa couplings also break the custodial symmetry and therefore modify the above prediction. Nevertheless, these effects arise at the one-loop order and give modifications of order $Y_t^2/(16\pi^2) \sim 0.01$ at most (Y_t is the top Yukawa coupling). Once more, an accidental symmetry, in this case not exact but approximate, gives a nontrivial prediction, $\rho \simeq 1$, that has been very well tested by experiments.

Other important approximate accidental symmetries of the SM are flavour symmetries. In the limit of $Y_{d,u,e} = 0$, the SM has five extra global $SU(3)$ symmetries associated to unitary transformations of the three families of Q_L^i , u_R^i , d_R^i , l_L^i and e_R^i respectively. These symmetries, or subgroups of them, allow us to explain a plethora of flavour physics properties, as you can find in the lectures of G. Hiller in this report.

All the accidental symmetries described above are symmetries of the SM due to the fact that only operators of dimension ≤ 4 are included in Eq. (3). As soon as we consider terms involving higher dimensional operators (terms having more fields or derivatives than those in Eq. (3)), these accidental symmetries are no longer preserved. For example, we can write the dimension-five operator

$$\mathcal{L}_{\text{SM}}^{(5)} = \frac{c_5}{\Lambda} \bar{l}_L^c \cdot H H \cdot l_L + h.c., \quad (7)$$

where $A \cdot B = \epsilon_{rs} A^r B^s$ with $r, s = 1, 2$ being $SU(2)_L$ indices, and \bar{l}_L^c is the conjugated of l_L . This term is suppressed by a mass scale Λ since the Lagrangian has to have dimension 4 (the coefficient c_5 is

²This symmetry is, however, ‘anomalous’, i.e. broken at the quantum level. This implies that the SM predicts that the proton should decay but at an extremely small rate.

³This is a $SU(2)$ rotation between H^r and $\epsilon^{rs} H_s$ where $r, s = 1, 2$ label the components of a $SU(2)_L$ doublet.

defined to be dimensionless). One can easily realize that the term in Eq. (7), present in principle for each SM lepton, violates the lepton symmetries. There are also operators of dimension 6 that violate L and B . In particular

$$\mathcal{L}_{\text{SM}}^{(6)} = \frac{c_6}{\Lambda^2} \epsilon_{rs} \epsilon_{\alpha\beta\gamma} [\bar{Q}_L^{c\tau\alpha} \gamma^\mu u_R^\beta] [\bar{d}_R^{c\gamma} \gamma_\mu l_L^s] + h.c. + \dots \quad (8)$$

where $\alpha, \beta, \gamma = 1, 2, 3$ run over the colour indices of $SU(3)_c$, $r, s = 1, 2$ are $SU(2)_L$ indices, and c_6 is a dimensionless coefficient that violates not only the lepton symmetries but also baryon number. If present in the SM, it will lead to proton decay. The experimental evidence of B and L conservation tells us that these terms must be either absent or be highly suppressed, i.e., $\Lambda \gg M_W$.

2.2 Consistency of the SM

There are other reasons for avoiding terms such as those of Eqs. (7) and (8) in the SM Lagrangian. We do not know how to quantize a field theory with operators of dimensions larger than 4. The best we can do in the presence of these higher dimensional operators is to assume that the SM is an effective field theory valid only up to energies $E \lesssim \Lambda$. Theorists call this scale Λ , that determines the energy below which a theory is valid, the ‘cutoff’ scale.

A question that immediately comes to mind is the following. Can we, in the SM, take $\Lambda \rightarrow \infty$? Or, in other words, is the SM a theory valid for all energies? We can try to answer this question either theoretically or experimentally. In the first case, we must check whether the SM predicts always consistent results. To do so, we perform Einstein’s Gedankenexperiment (thought experiment) looking for inconsistencies of the SM, similar to those that led Einstein to postulate the theory of Special Relativity in order to reconcile the laws of Newtonian classical mechanics with the laws of electromagnetism. Experimentally, we can also address the above question by asking whether the SM Lagrangian of Eq. (3) explains all present experimental data. If not, an extension of it will be needed implying that the SM has a finite cutoff scale Λ above which new physics shows up.

Let us first try to answer the above question from the theoretical perspective. The Gedankenexperiment that we proposed here, in order to check the validity of the SM, is to calculate the amplitude of Higgs scattering, $hh \rightarrow hh$, at very high energies. At tree level this amplitude is proportional to λ . At the quantum level this amplitude is well approximated by using the ‘running coupling’ $\lambda(Q)$, where the scale Q can be approximately associated with the energy at the centre of mass of the process. The running coupling $\lambda(Q)$ can be easily obtained by solving the renormalization group equation (RGE):

$$\frac{d\lambda(Q)}{d \ln Q} = \frac{1}{16\pi^2} (24\lambda^2 + 12\lambda Y_t^2 - 6Y_t^4) + \dots, \quad (9)$$

where we only show the dominant one-loop result arising from the top and Higgs. As we increase Q , $\lambda(Q)$ can increase or decrease, depending on its initial value, taken, for example, at $Q = 10^3$ GeV. This is shown in Fig. 1. If $\lambda(Q)$ grows with Q , we can reach a point at which $\lambda(Q)$ is too large and we cannot calculate within perturbation theory. Let us call the scale at which this happens Λ_+ . At $Q = \Lambda_+$ the SM becomes intractable, and even lattice studies (used to study theories with large couplings such as QCD) have shown that the SM cannot take such initial values of λ if at the same time we demand $\Lambda_+ \rightarrow \infty$. In other words, for those initial values of λ for which $d\lambda(Q)/d \ln Q > 0$, the SM is only valid up to energies $\sim \Lambda_+$, i.e., the SM has a nonzero cutoff scale $\Lambda \sim \Lambda_+$. On the other hand, if $\lambda(Q)$ decreases with Q , it can become negative at some $Q = \Lambda_-$ in which case the electroweak vacuum is only a local minimum and there is a new deeper and potentially dangerous minimum at this scale. Up to some caveats discussed in Ref. [4], the SM cannot be valid at energies at which $\lambda(Q)$ is negative, showing again a limitation in the SM, $\Lambda \sim \Lambda_-$. By relating λ with the Higgs mass, $M_h = \sqrt{2\lambda(Q \sim M_h)}v$, we show in Fig. 2 the range of validity of the SM (the scale Λ) versus M_h . We can see that there is only a small window around $M_h \sim 150$ GeV in which the SM is valid, at least, up to $Q \sim 10^{19}$ GeV. Why do we stop at this scale? Because at energies above 10^{19} GeV, gravity becomes important and must be

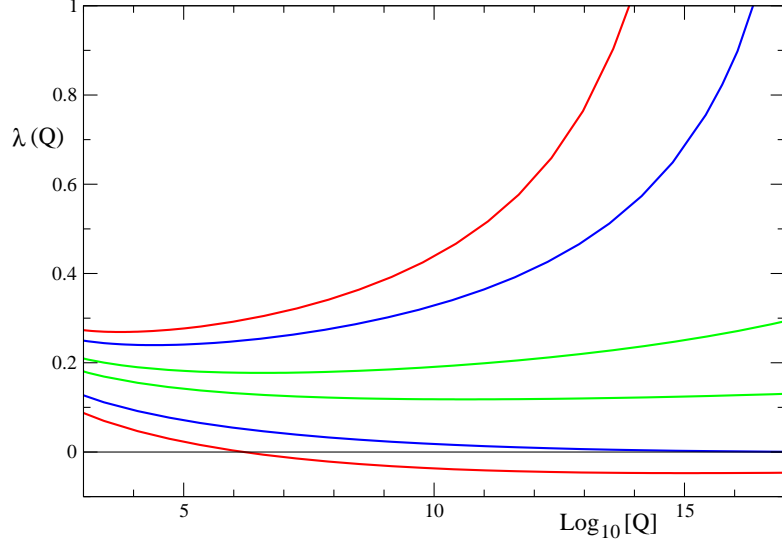


Fig. 1: Value of $\lambda(Q)$ as a function of Q for different initial values of $\lambda(Q = 10^3 \text{ GeV})$

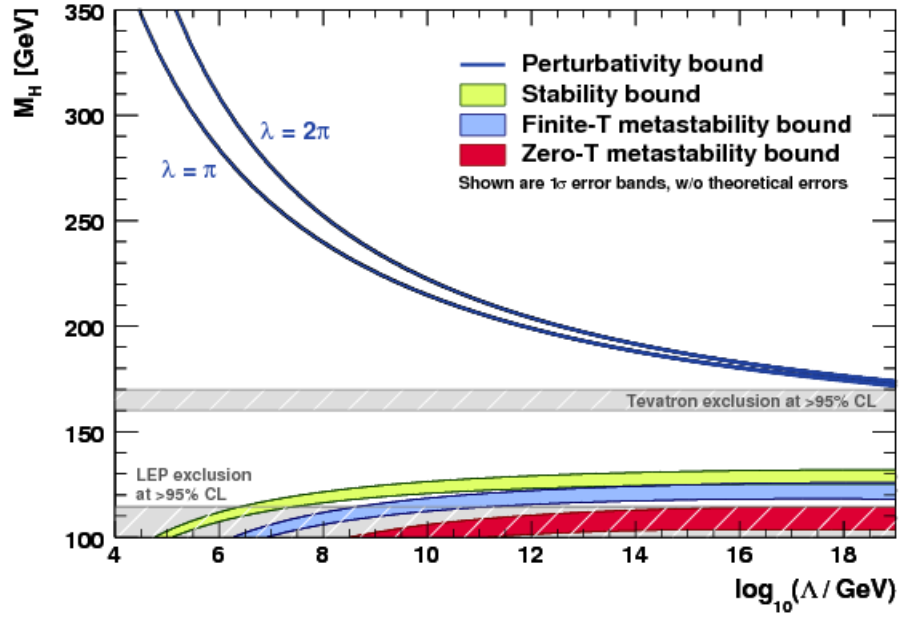


Fig. 2: Values of M_h as a function of Λ . See details in Ref. [4].

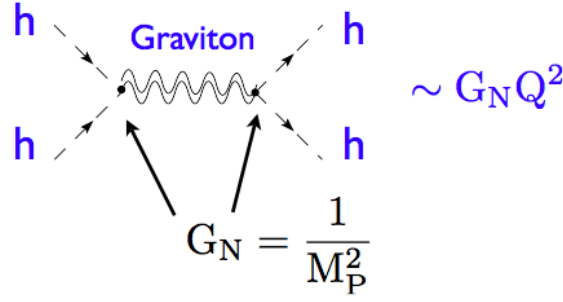


Fig. 3: Feynmann diagram for the contribution of a graviton to the $hh \rightarrow hh$ process

considered in the calculation. The new contribution, given in Fig. 3, grows with Q as $G_N Q^2$, where G_N is the Newton constant, and becomes relevant at energies $\sim \sqrt{1/G_N} \equiv M_P \simeq 10^{19}$ GeV. We do not know how to treat at the quantum level a theory of gravity at energies above the Planck scale M_P . For this reason the SM has an ultimate cutoff⁴ scale $\Lambda \sim M_P$. Above this scale we do not know how to calculate physical quantities. It is interesting to compare this situation with Fermi's theory. When Fermi proposed his theory for the weak interactions it was clear for him that this was a theory valid up to energies $\sim \sqrt{1/G_F} \simeq 300$ GeV. He was right, and we know today that at energies around $\sqrt{1/G_F}$ there is 'new physics' beyond Fermi theory, the W and the Z gauge bosons. Similarly, we expect that at energies around $\Lambda \sim M_P$ new physics beyond the SM will show up. One possibility for this new physics is string theory that consists in replacing fields and particles by strings. The SM particles would correspond to massless string excitations, while massive excitations will have masses of order M_P .

Let us now move to the experimental consistency of the SM. Is there any experiment that cannot be explained by the SM? We find four pieces of experimental evidence beyond the SM:

1. Neutrino oscillations that require that neutrinos be massive.
2. The need for dark matter in the Universe.
3. The presence of a cosmological inflationary epoch.
4. The matter-antimatter asymmetry in the Universe.

Indeed, the SM predicts that neutrinos are massless, does not have a candidate for dark matter, and a cosmological inflationary epoch or a matter–antimatter asymmetry cannot be produced at the desirable rate. Although physics beyond the SM is required, it is important to be aware of the fact that all the above experimental evidence beyond the SM does not really require new physics at scales much below M_P . Then, since as explained above the presence of gravity tells us that the SM is an effective theory with a maximum cutoff scale around M_P , we can expect Planckian physics to be responsible for the above experimental disagreements. For example, if the SM is not valid above Λ , we can expect terms as those of Eqs. (7) and (8) to be present in the theory. After electroweak symmetry breaking (EWSB), Eq. (7) leads to neutrino masses of order

$$m_\nu \sim \frac{v^2}{\Lambda} \sim 0.06 \text{ eV} \left(\frac{10^{15} \text{ GeV}}{\Lambda} \right). \quad (10)$$

Therefore neutrino masses can be induced with the right magnitude for a Λ not far away from M_P . Also the other experimental conflicts can be blamed on physics at around the Planck scale, so one can maintain that experiments do not really provide any evidence for a lower cutoff scale in the SM.

⁴One can check that the other SM interactions are always well behaved at energies below M_P .

2.3 Reasons for improvement in the SM

So far we have seen that the SM can be a theory valid all the way to energy scales around the Planck scale. Experimental or theoretical inconsistencies are not enough to put the SM in trouble at lower energies. There are, however, other theoretical reasons to go beyond the SM: the search for a ‘natural’ explanation of the SM parameters. Here we list some of them, from the most important one to the less important according to my taste:

1. *The cosmological constant:*

Experimentally we know that it takes the value $\Lambda_{cosmo} \sim 10^{-47} \text{ GeV}^4$.

Theoretically we would have expected $\Lambda_{cosmo} \sim \Lambda^4 \sim M_P^4 \sim 10^{76} \text{ GeV}^4$.

2. *The Higgs mass term:*

In order to give the right Higgs VEV, we must have $\mu^2 \sim v^2 \sim 10^4 \text{ GeV}^2$.

Theoretically one would have expected $\mu^2 \sim \Lambda^2 \sim M_P^2 \sim 10^{38} \text{ GeV}^2$.

3. *Charge quantization:*

We do not have any explanation in the SM of why the electron charge is equal but opposite in sign to the proton charge, as experiments suggest to us: $Q_e + Q_p < 10^{-21}$.

4. *The strong CP problem:*

We do not understand why in the SM the term $\int d^4x \theta \epsilon^{\mu\nu\rho\sigma} G_{\mu\nu}^a G_{\rho\sigma}^a$, that would lead to CP violation in the strong sector, has a coefficient θ so small. Experimentally we know that $\theta \lesssim 10^{-13}$.

5. *Fermion masses and mixing angles:*

The fermion mass spectrum ranges from $\sim 170 \text{ GeV}$, for the case of the top-quark, to $\sim 10^{-3} \text{ GeV}$, for the case of the electron. We do not know why there exists such a large difference in masses. We also find experimentally that the Cabibbo–Kobayashi–Maskawa matrix is very close to a diagonal matrix. We do not know why.

6. *Gauge coupling unification:*

We find experimentally that $g_s \sim 1.12$, $g \sim 0.65$, and $g' \sim 0.35$ at $E \sim M_Z$. Is there any reason for such differences?

7. *Number of families:*

Matter is made of three families. Is there any reason for this?

Let us briefly comment on them. We do not have any natural explanation for the cosmological constant value; it is still a true mystery to us. We do, however, have several possible explanations for the smallness of the Higgs mass as compared to M_P (usually referred as the hierarchy problem). Below we will discuss the most interesting ones, supersymmetry, Higgsless models, composite Higgs, and extra dimensions. On charge quantization and gauge coupling, unification theorists have postulated the existence of Grand Unified Theories at high energies that could explain these relations. We will discuss them below. For the strong CP problem, we have a nice explanation with a nice prediction, the axion state. We will devote Section 4 to it. Finally, several explanations for the observed fermion masses, mixings, and the number of families have been proposed in the literature in the last years. I do not find any of them compelling enough to single them out here. The usual problem with models of fermion masses is that they do not lead to sharp predictions.

3 Grand Unified Theories (GUT) [5]

If we open the 2010 edition of the Particle Data Group [1] we find

$$|Q_e + Q_p|/e < 1.0 \times 10^{-21}. \quad (11)$$

This strong constraint suggests that the electric and proton charge are quantized following the relation

$$Q_e = -Q_p. \quad (12)$$

$$\bar{5} = \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \\ e^- \\ -\nu_e \end{pmatrix} \quad 10 = \begin{pmatrix} 0 & \begin{matrix} u_3^c & -u_2^c \end{matrix} & \begin{matrix} -u_1 & -d_1 \\ -u_2 & -d_2 \\ -u_3 & -d_3 \end{matrix} \\ \begin{matrix} 0 & u_1^c \end{matrix} & 0 & \begin{matrix} 0 & -e^c \\ 0 & 0 \end{matrix} \end{pmatrix}$$

Fig. 4: The embedding of a family of SM fermions. The indices 1, 2, 3 refer to the three colours. The fields d^c , u^c , and e^c are the conjugated of d_R , u_R , and e_R respectively.

Since in the SM we have $Q = Y/2 + T_3$, Eq. (12) implies that the hypercharges are quantized following the relations

$$Y_{l_L} = 2Y_{e_R} = -\frac{4}{3}Y_{u_R} = \frac{2}{3}Y_{d_R} = -\frac{1}{3}Y_{Q_L}. \quad (13)$$

In the SM, since the $U(1)_Y$ is an Abelian group, the hypercharges could have been, in principle, arbitrary real numbers. It is then surprising to find the relation Eq. (13)⁵.

A possible explanation for the SM hypercharge quantization comes from assuming that the SM group of symmetries is, at high energies, a much larger group G . If this group G only contains non-Abelian groups all charges will be quantized. The minimal group G fulfilling this requirement is the Pati–Salam group [6] $SU(4) \times SU(2)_L \times SU(2)_R$. Demanding G to be a simple group, the minimal case is $G = SU(5)$, the model of Georgi and Glashow [7]. We discuss it next.

3.1 $SU(5)$ GUT

The $SU(5)$ group is defined as the set of 5×5 unitary matrices with determinant 1. This group contains $SU(3) \times SU(2) \times U(1)$ as a subgroup, corresponding respectively to the 5×5 matrices

$$\left(\begin{array}{c|c} U_{3 \times 3} & \\ \hline & 0 \end{array} \right), \quad \left(\begin{array}{c|c} 0 & \\ \hline & U_{2 \times 2} \end{array} \right), \quad \begin{pmatrix} e^{i\frac{2}{3}\theta} & & & & \\ & e^{i\frac{2}{3}\theta} & & & \\ & & e^{i\frac{2}{3}\theta} & & \\ & & & e^{-i\theta} & \\ & & & & e^{-i\theta} \end{pmatrix}, \quad (14)$$

where $U_{3 \times 3}$ ($U_{2 \times 2}$) is a 3×3 (2×2) matrix. The last matrix, to be associated with a $U(1)_Y$ transformation, shows that, as expected, the hypercharges are no longer free numbers but they are quantized. The $SU(5)$ group has 24 generators, each of them has an associated gauge boson. Only 12 of them can be identified with the SM gauge bosons. The other 12 are extra gauge bosons that must get masses M_{GUT} above the electroweak scale where the $SU(5)$ must be broken. These extra gauge bosons, referred to as X and Y bosons, have charges $(\mathbf{3}, \mathbf{2})_{5/3}$ and $(\bar{\mathbf{3}}, \mathbf{2})_{-5/3}$ under the SM group.

The SM fermions must be embedded in $SU(5)$ representations. Amazingly, Georgi and Glashow realized that a full SM family of fermions could be neatly embedded into two $SU(5)$ representations, the $\bar{5}$ and the 10 . The explicit embeddings are given in Fig. 4. These embeddings give the correct hypercharge assignments Eq. (13). Such simplicity, however, does not occur in the embedding of the Higgs doublet into an $SU(5)$ representation. The minimal case is to embed the Higgs into a $\bar{5}$, but this requires one to introduce a colour triplet accompanying the Higgs. Similarly to the X, Y bosons, this colour triplet must get a mass when $SU(5)$ is broken. This is known as the doublet-triplet splitting problem in GUT.

⁵We must say, however, that the SM hypercharges are not really free parameters since the absence of quantum anomalies in the SM forces them to fulfil a set of equations. Equation (13) is a particular case that leads to an anomaly-free theory.

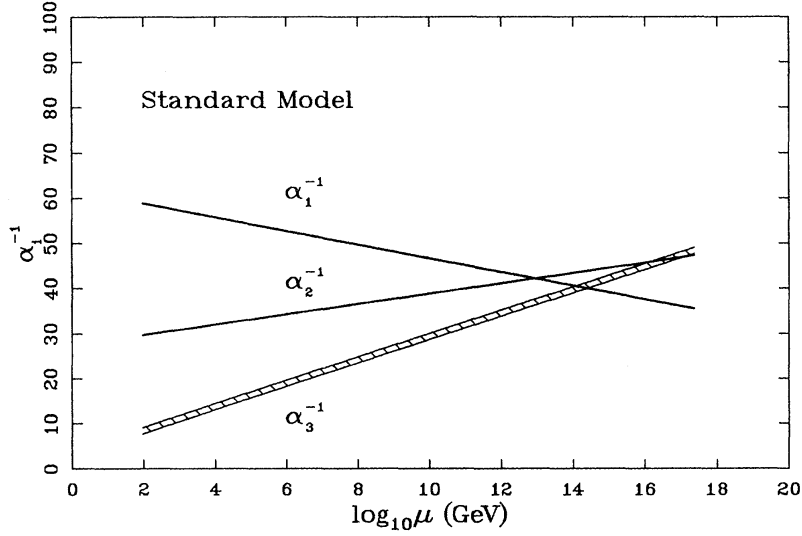


Fig. 5: Evolution of the three SM gauge couplings $\alpha_i = g_i^2/(4\pi)$ as a function of $\mu = Q$ in the SM [8]

The $SU(5)$ model gives us three interesting predictions:

1. Hypercharge quantization.
2. Gauge coupling unification.
3. Proton decay.

We already mentioned the first one. Let us comment on the second one. If the $SU(5)$ symmetry is exact we have that all SM gauge couplings must be equal:

$$g_s = g = \sqrt{\frac{5}{3}}g' \equiv g_5, \quad (15)$$

where the factor $\sqrt{\frac{5}{3}}$ arises from the proper normalization of g' . Nevertheless, if the $SU(5)$ symmetry is broken at some scale M_{GUT} we only expect Eq. (15) to be fulfilled at energies above M_{GUT} . Indeed, in a quantum field theory the gauge couplings ‘runs’ with the energy according to the RGE. At the one-loop level we have

$$\frac{dg_i}{d \ln Q} = -\frac{b_i}{8\pi^2}, \quad (16)$$

where $g_3 = g_s$, $g_2 = g$, $g_1 = \sqrt{\frac{5}{3}}g'$ and b_i are coefficients that depend on the spectrum of the theory. Above M_{GUT} the spectrum of particles corresponds to that of a $SU(5)$ theory and we have $b_1 = b_2 = b_3$, but below M_{GUT} the X, Y states and the colour partner of the Higgs are not present. The b_i are only sensitive to the SM spectrum; we have $b_i = (41/10, -19/6, -7)$. In Fig. 5 we plot the evolution of the three SM gauge couplings $\alpha_i = g_i^2/(4\pi)$ as a function of Q . We see that the gauge couplings tend to unify at energies around 10^{14} GeV, although Eq. (15) is not precisely satisfied. One could argue that this is a small discrepancy, originating from high-energy corrections to the gauge couplings. Even so, this implies $M_{\text{GUT}} \sim 10^{14}$ GeV and, as we will see later, a conflict with proton decay experiments. A better situation occurs in the supersymmetric SM that we will introduce later motivated by the hierarchy problem. In this model we have $b_i = (66/10, 1, -3)$ and a different evolution of the gauge couplings as compared with the SM, as shown in Fig. 6. Now the three SM gauge couplings neatly unify at energies

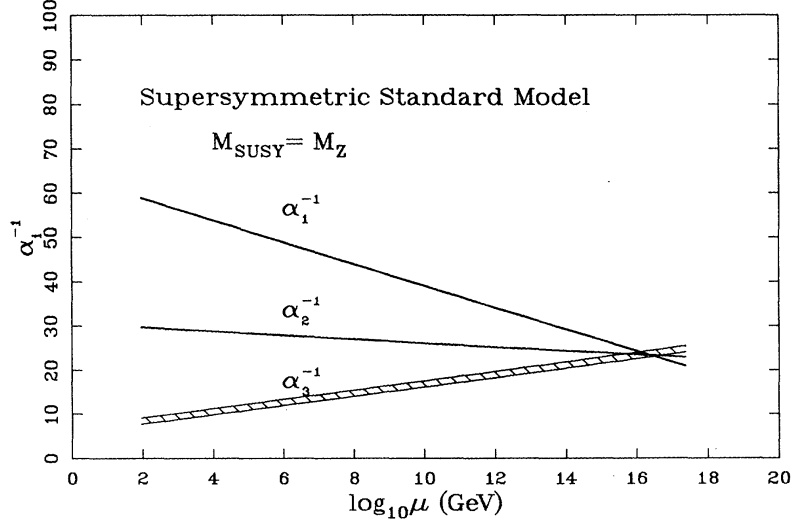


Fig. 6: Evolution of the three SM gauge couplings $\alpha_i = g_i^2/(4\pi)$ as a function of $\mu = Q$ in the Supersymmetric SM [8]

$\sim 10^{16}$ GeV, the scale to be associated with M_{GUT} .

Let us finally comment on proton decay. In the $SU(5)$ model the baryon symmetry is not preserved. This is obvious since we have put quarks and leptons in the same representation — see Fig. 4. Therefore we expect to have contributions to proton decay. We can explicitly see that this decay is mediated by the X and Y bosons that generate the operator of Eq. (8) with $\Lambda \sim M_{\text{GUT}}$. We obtain

$$\tau(p \rightarrow \pi^0 e^+) \sim 10^{34} \text{ years} \left(\frac{3 \times 10^{15} \text{ GeV}}{M_{\text{GUT}}} \right)^4. \quad (17)$$

The Super-Kamiokande detector 1000 metre underground in the Kamioka mine of Hida city (Gifu) Japan, has the ‘titanic’ task of searching for proton decay. This is a stainless-steel tank 39 m in diameter and 42 m tall. It is filled with 50 000 tons of ultra pure water and about 13,000 photomultipliers are placed on the tank wall. It looks for pions and positrons arising from the proton decay of the water. Neutral pions decay to photons that can be detected by the photomultipliers, while positrons travelling through the water emit Cherenkov light that can also be detected by the photomultipliers. At present they put a bound of $\tau(p \rightarrow \pi^0 e^+) > 10^{34}$ years corresponding, according to Eq. (17), to the bound $M_{\text{GUT}} > 3 \times 10^{15}$ GeV. This rules out $SU(5)$ models with $M_{\text{GUT}} \sim 10^{14}$ GeV, and is at the verge of testing models, such as supersymmetric $SU(5)$ models⁶, where $M_{\text{GUT}} \sim 10^{16}$ GeV.

Apart from the three predictions explained above, GUT give other type of interesting predictions, although they are more model dependent. For example in most of GUT bottom-tau unification is predicted: $M_b = M_\tau$ at $Q \gtrsim M_{\text{GUT}}$. This prediction works reasonable well in the supersymmetric SM. Nevertheless it does not work for the other families. Another prediction of GUT with $G = SO(10)$ is the generation of neutrino masses through the ‘see-saw’ mechanism. In $SO(10)$ all SM fermions of a given family can be embedded in a single representation, the 16 of $SO(10)$. Apart from the SM fermions it also contains a singlet ν_R that after $SO(10)$ breaking can get a mass and generate the operator of Eq. (7) with $\Lambda \sim M_{\nu_R}$. We already saw that this operator leads to neutrino masses of the Majorana type. This also

⁶In supersymmetric $SU(5)$ models we have other proton decay channels, e.g., $p \rightarrow K^+ \bar{\nu}_\tau$, that are usually more important than the one considered here [5].

leads to processes with neutrinoless double beta-decays: $nn \rightarrow ppee$. Thus, observing experimentally this process is of great importance for understanding the nature of the neutrino masses.

4 The strong CP-problem and axions

In the SM Lagrangian of Eq. (3) we did not include the dimension-4 operator $\epsilon^{\mu\nu\rho\sigma} G_{\mu\nu}^a G_{\rho\sigma}^a$ made of gluon fields⁷. Following the usual convention in the literature, let us introduce it as

$$\frac{\theta}{32\pi^2} \int d^4x \epsilon^{\mu\nu\rho\sigma} G_{\mu\nu}^a G_{\rho\sigma}^a. \quad (18)$$

This term violates the CP symmetry and induces a sizeable electric dipole moment (EDM) for the neutron. Experimental limits on the neutron EDM put a limit on the coefficient θ :

$$\theta \lesssim 10^{-10}. \quad (19)$$

The smallness of this coefficient requires an explanation. A possible one was proposed long ago by Peccei and Quinn [9]. They promoted θ to a field $a(x)$, the axion field, assumed to be a Goldstone boson arising from the spontaneous breaking of a $U(1)$ symmetry, the PQ symmetry. If this symmetry had a $U(1)SU(3)_c^2$ -anomaly, then the only non-derivative interaction of the axion would be given by the term of Eq. (18) with the replacement $\theta \rightarrow a(x)/f_A$, where f_A is a dimensionful parameter called the axion decay-constant. In this model the value of θ is dynamical and must be calculated by minimizing the axion potential. One obtains $V(a) = \frac{1}{2}m_A^2 a(x)^2 + \dots$ and then, $\langle a(x) \rangle = 0 \Rightarrow \theta = 0$, in agreement with Eq. (19).

The Peccei–Quinn mechanism has a testable prediction [10]. The model predicts the existence of a new particle, the axion, whose mass can be calculated. In the limit $f_A \gg f_\pi$, we have

$$m_A = \frac{f_\pi}{f_A} \frac{\sqrt{m_u m_d}}{m_u + m_d} m_\pi. \quad (20)$$

The axion mass ranges from 100 keV to 10^{-12} eV, as the unknown parameter f_A varies from 100 GeV to 10^{19} GeV. Axions couple to gluons through Eq. (18) with $\theta \rightarrow a(x)/f_A$, so the larger f_A , the smaller are their couplings to SM states. Detecting the axion would be an excellent way to prove the Peccei–Quinn idea. Since the proposal of this mechanism, experimentalists have been searching in vain for the axion. Today the values of f_A (or equivalently of m_A) are strongly constrained, as shown in Fig. 7.

The axion field is a possible dark matter candidate if f_A lies around 10^{12} GeV. The ADMX experiment is looking for dark matter axions coming from the halo of the galaxy. Since the axion couples to gluons, it mixes with the pions, and since these latter couple to photons, the axions also, generically, couple to photons. Then axions can scatter off a magnetic field and resonantly be converted into microwave photons. The present searches at ADMX are aiming axions with f_A between 10^{11} GeV and 10^{13} GeV as shown in Fig. 7.

5 The hierarchy problem

In the SM the electroweak symmetry is triggered by the Higgs VEV. For positive values of μ^2 , the Higgs VEV is given by $v^2 = \mu^2/\lambda$. Therefore we must have $\mu^2 = \lambda v^2 \sim 6 \times 10^4 \lambda \text{ GeV}^2$. As compared with the other dimensionful scale of the SM, the Planck scale, $M_P^2 \sim 10^{38} \text{ GeV}^2$, the value of μ^2 looks extremely small. Why are they so different? This is the so-called hierarchy problem.

To make it worse, one can realize that the Higgs mass is, at the quantum level, very sensitive to the mass of heavy states to which the Higgs couples. For example, in the $SU(5)$ GUT discussed above, the

⁷Also the same operator but made of W_μ could be present. The impact of this term, however, on physical observables is negligible.

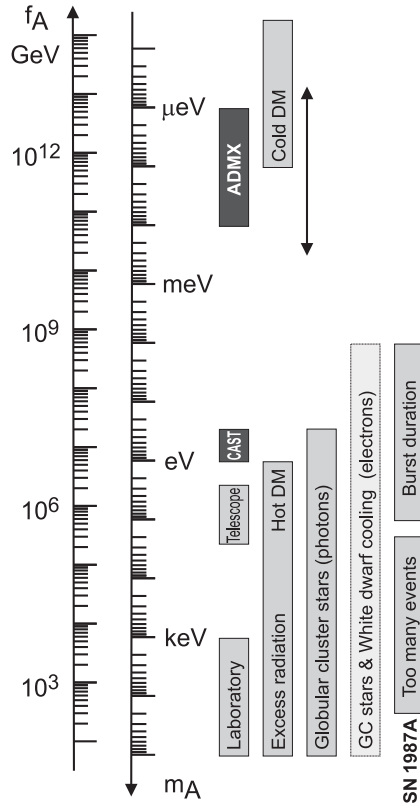


Fig. 7: Excluded regions of f_A (or equivalently of m_A) from different experiments and astrophysical constraints. The reach of present experiments CAST and ADMX is also shown [1].

Higgs couples to the X and Y bosons. At the one-loop, the Higgs mass squared will receive corrections proportional to $M_{\text{GUT}}^2/(16\pi^2)$. Therefore it is very unnatural to expect $\mu^2 \ll M_{\text{GUT}}^2$.

This sensitivity to the heavy states is a feature only of scalars. For fermions, for example, we have that one-loop corrections to a Dirac fermion mass are proportional to the fermion mass itself, and similarly for gauge bosons. The reason for this can be easily understood using symmetries. A Dirac fermion mass arises from the operator $m\bar{\psi}_L\psi_R$. This operator is not invariant under the chiral phase transformation $\psi_R \rightarrow e^{i\theta}\psi_R$. This means that it can only be generated from terms in the Lagrangian that break this symmetry. Heavy states respecting the chiral symmetry cannot induce it. We say that the fermion masses are ‘protected’ by chiral symmetries. Similarly for gauge bosons, the gauge symmetry protects the mass of the gauge bosons. On the other hand, scalar mass terms, e.g., $\mu^2|H|^2$, are invariant under any phase transformation and then can be induced by heavy virtual particles.

Three solutions have been proposed to solve the hierarchy problem: (1) Implement a symmetry that relates scalars to fermions since the masses of these latter are not sensitive to heavy states. This is supersymmetry. (2) Assume that the Higgs is not elementary but just a composite state. (3) Assume that the only scale in particle physics is the electroweak scale, e.g., $v \simeq 246$ GeV. In this case M_P is not the high-energy scale at which gravity becomes strong, but just an unphysical scale arising from the fact that the Newton constant, that on dimensional grounds is now given by $G_N = g_N/v^2$, is extremely small at large distances, $g_N \ll 1$. This can be naturally realized assuming the existence of extra dimensions. We will discuss all of them in turn.

6 Supersymmetry

Supersymmetry provides a symmetry that protects the Higgs mass. It works in the following way. Supersymmetry relates scalars to fermions; since the masses of these latter are protected by (chiral) symmetries, scalar masses will also be protected. An instructive way to see this is by looking at the simplest case, a free theory of a Majorana fermion Ψ and a complex scalar Φ . Its Lagrangian is given by

$$\mathcal{L} = |\partial_\mu \Phi|^2 + i \frac{1}{2} \bar{\Psi} \not{\partial} \Psi. \quad (21)$$

This Lagrangian is invariant under

$$\begin{aligned} \Phi &\rightarrow \Phi + \delta\Phi & \delta\Phi &= \bar{\xi}(1 - \gamma_5)\Psi \\ \Psi &\rightarrow \Psi + \delta\Psi & \delta\Psi &= i(1 - \gamma_5)\gamma^\mu \xi \partial_\mu \Phi, \end{aligned} \quad (22)$$

where ξ , the parameter of the transformation, is a Majorana fermion (anticommuting). Note that a mass for the scalar, $\mu^2|\Phi|^2$, is not invariant under the symmetry Eq. (22). In other words, this symmetry forbids the scalar to get a mass. Equation (22) is a supersymmetry. It can be shown that supersymmetry is the maximal extension of the Poincaré group in a quantum field theory [11]. It contains an extra generator Q that acting on fermionic states transforms them into bosonic states and vice versa⁸. In a schematic form the SuperPoincaré algebra is given by

$$\begin{aligned} [Q, M_{\mu\nu}] &= Q, \\ \{Q, Q^\dagger\} &= P^\mu, \\ \{Q, Q\} &= \{Q^\dagger, Q^\dagger\} = 0, \\ [P^\mu, Q] &= [P^\mu, Q^\dagger] = 0. \end{aligned} \quad (23)$$

The Q generator commutes with P^2 and any generator of the gauge group. This implies that a fermion and its associated boson have equal mass and charge.

Imposing supersymmetry on the Standard Model leads to the Minimal Supersymmetric Standard Model (MSSM). This is not a straightforward exercise, and we redirect the interested reader to Ref. [12]. Here we will only comment on the most important implications of supersymmetry and its predictions at present and future colliders.

The most drastic implication of supersymmetry is that the SM spectrum is required to be doubled. For each SM quark and lepton (Q_L, l_L, \dots) one has to add an extra scalar, usually called squark and slepton ($\tilde{Q}_L, \tilde{l}_L, \dots$), and for each gauge boson and Higgs (W_μ, H, \dots) one has to add an extra fermion, called gauginos and Higgsinos ($\tilde{W}, \tilde{H}, \dots$). But this is not yet enough. This theory will have anomalies (quantum inconsistencies) that, to be avoided, require the addition of extra fields. The simplest way is to add an extra Higgs doublet. The Higgs sector is then given by

	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$
H_u	1	2	1
H_d	1	2	-1

(24)

that are accompanied by two Higgsinos, \tilde{H}_u and \tilde{H}_d , with the same quantum numbers. In the MSSM not all possible terms allowed by symmetries can be added. Some of them would lead to a violation of the baryon and lepton symmetries, in clear contradiction with experiments. An easy way to avoid these terms is to impose a discrete symmetry on the MSSM, under which all SM fields are *even* and

⁸The supersymmetry considered here is often called $\mathcal{N} = 1$ supersymmetry. There exist extensions to this supersymmetry ($\mathcal{N} = 2, 4$) with a more extended algebra. They will not be discussed here since there is no phenomenological motivation for these extensions.

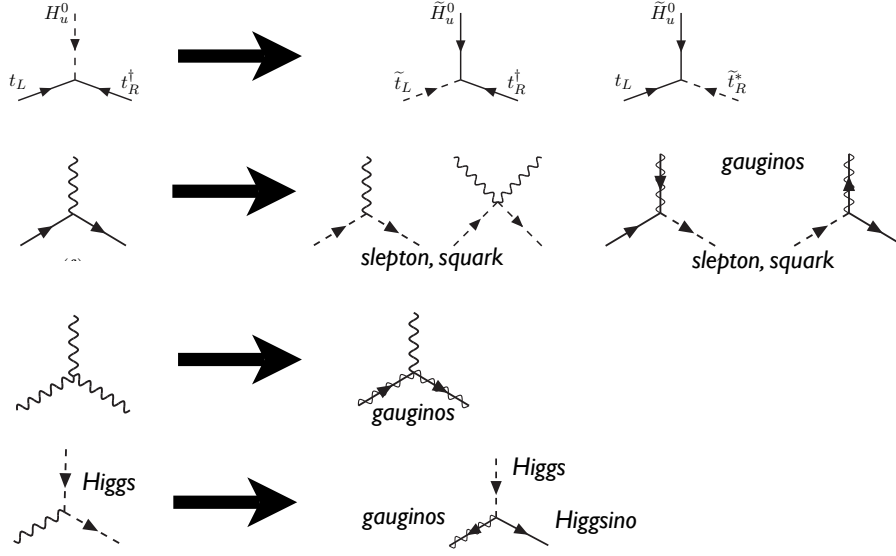


Fig. 8: MSSM interactions obtained from ‘supersymmetrization’ of the SM interactions

all superpartners are *odd*. This is an R -parity. Imposing this discrete symmetry on the MSSM leads to interesting consequences. The superpartners can only be produced in pairs; the lightest supersymmetric particle (LSP) is stable, resulting in a dark matter candidate. Furthermore, the Yukawa couplings take the form

$$\mathcal{L}_Y = Y_u^{ij} \bar{Q}_L^i \cdot H_u^* u_R^j + Y_d^{ij} \bar{Q}_L^i \cdot H_d^* d_R^j + Y_e^{ij} \bar{l}_L^i \cdot H_d^* e_R^j + h.c. \quad (25)$$

The Yukawa couplings then become very sensitive to the ratio of the Higgs VEVs, $\langle H_u \rangle / \langle H_d \rangle \equiv \tan \beta$, since we have

$$Y_u = \frac{g M_u}{\sqrt{2} M_W \sin \beta}, \quad Y_{d,e} = \frac{g M_{d,e}}{\sqrt{2} M_W \cos \beta}. \quad (26)$$

Although the derivation of all MSSM interactions is a difficult task, it is relatively easy to obtain the main interactions needed for phenomenology. They can be obtained by ‘supersymmetrization’ that corresponds to taking any SM interaction and replacing $fermion \leftrightarrow boson$ consistently with the SM symmetries. This is depicted in Fig. 8. The only interactions not obtained in this way are scalar trilinears and quartics.

If supersymmetry is exact, the mass of a fermion and that of its associated boson must be the same. This implies that, for example, s-electrons must have mass of half MeV. We have not seen such a light state, implying that supersymmetry must be broken. Like in the SM with the electroweak symmetry, we can assume that supersymmetry is spontaneously broken such that all superpartners receive mass. These masses cannot be much larger than the electroweak scale, otherwise we will have back the hierarchy problem that was the main motivation for supersymmetry. Indeed, after supersymmetry breaking, corrections to the Higgs masses become proportional to the superpartner masses; these latter must then be kept around the electroweak scale. Several realistic models of supersymmetry breaking have been proposed in the literature. The simplest one is called Gauge Mediated Supersymmetry Breaking (GMSB) [13]. It requires an extra sector responsible for spontaneous breaking of supersymmetry, containing fields charged under the SM gauge group, the ‘messengers’. The MSSM only knows about supersymmetry breaking due to the gauge interactions; therefore gaugino, squark, and slepton masses arise

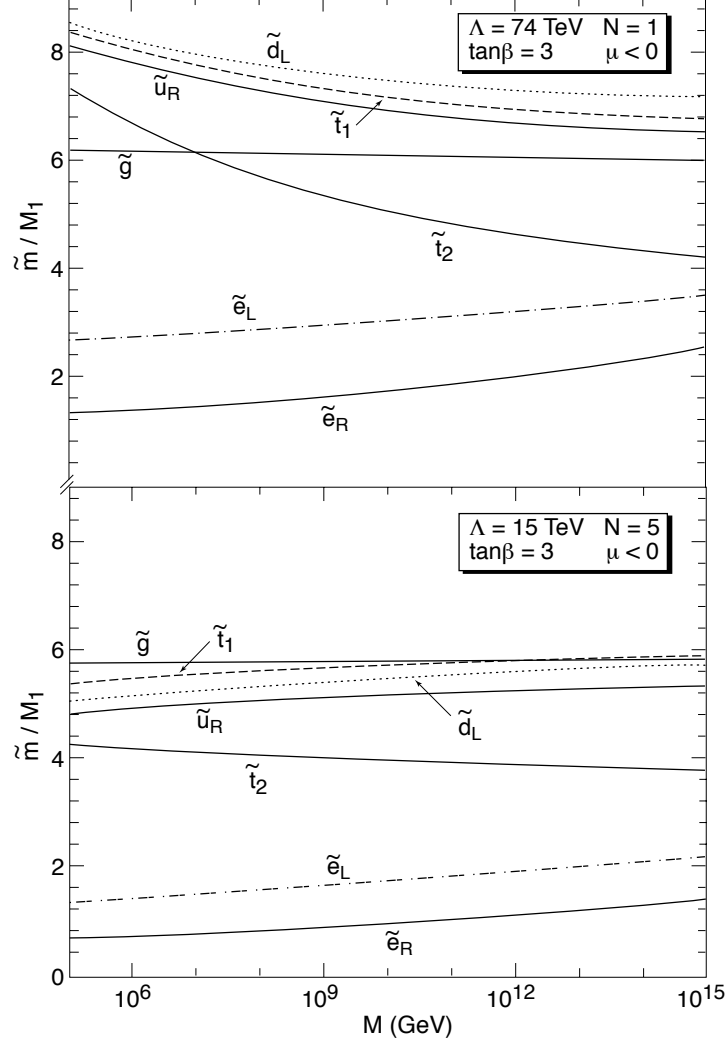


Fig. 9: Superparticle mass spectrum, normalized to the bino mass M_1 , in GMSB models taken from Ref. [13]. Here $\Lambda = F/M$ and N is the number of messengers.

at the loop level from these interactions that mediate the breaking from the supersymmetry-breaking sector to the MSSM sector. The model is quite predictive. Up to EWSB effects, gaugino and scalar masses depend on only two parameters: the supersymmetric breaking scale \sqrt{F} and the mass of the messengers M . The scalar masses are the same for all families, guaranteeing the absence of dangerous flavour-violating interactions. This is a crucial requirement to obtain realistic scenarios of supersymmetry breaking. The generation of the Higgsino mass is a difficult task in GMSB models and requires an extension of the model [14]. After EWSB, the spectrum is (slightly) modified and becomes sensitive to $\tan \beta$. A typical spectrum is shown in Fig. 9. The colour states are the heaviest, while right-handed sleptons, that have only hypercharge interactions, and binos \tilde{B} , are the lightest. The LSP, however, not shown in Fig. 9, is the superpartner of the graviton, the gravitino. Its mass is given by

$$m_{3/2} = \frac{F}{k\sqrt{24\pi}M_P} = \frac{1}{k} \left(\frac{\sqrt{F}}{100 \text{ TeV}} \right)^2 2.4 \text{ eV} , \quad (27)$$

where k is a model-dependent coefficient such that $k < 1$.

Another popular scenario of supersymmetry breaking is the so-called minimal supergravity model or constrained MSSM [15]. It is usually presented as a model that predicts, at energies $Q \simeq M_{\text{GUT}} \sim$

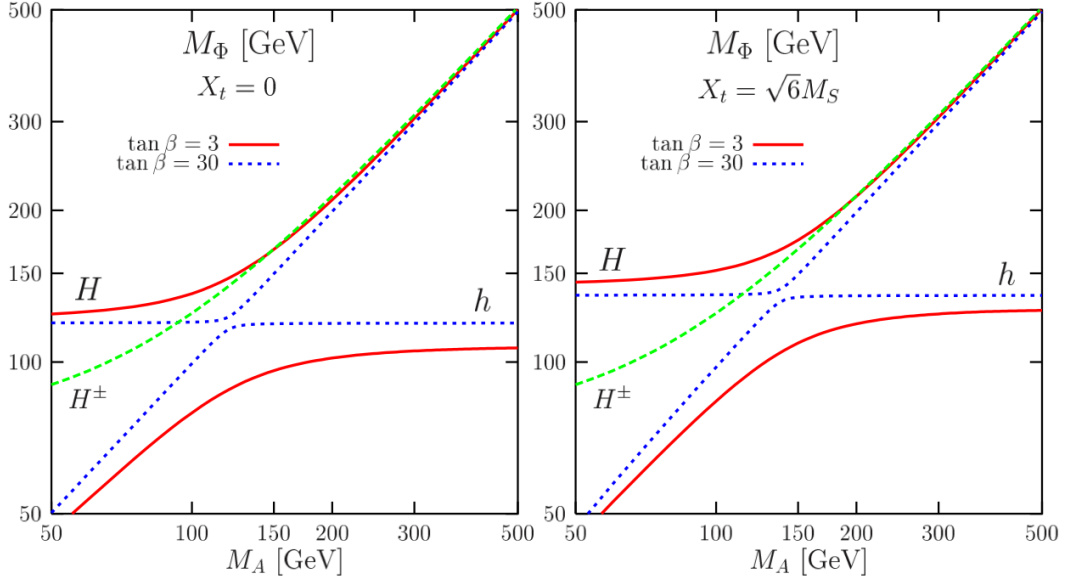


Fig. 10: Higgs spectrum in the MSSM taken from Ref. [16].

10^{16} GeV, universal gaugino masses, $M_{1/2}$, universal scalar masses, M_0 , and universal trilinears, A_0 . These ‘predictions’ however are not justified by any symmetry. Therefore one must consider this scenario just as a simplified Ansatz on the MSSM parameters, and not as a model. The LSP in this scenario is the neutralino, $\tilde{\chi}^0$, that is a mixture of neutral gauginos and Higgsinos.

A quite constrained sector of the MSSM is the Higgs sector. The Higgs potential is given by

$$\begin{aligned}
 V(H_u, H_d) &= m_1^2 |H_u|^2 + m_2^2 |H_d|^2 + m_{12}^2 H_u \cdot H_d + h.c. \\
 &+ \frac{g^2 + g'^2}{8} (|H_u|^2 - |H_d|^2)^2 + \frac{g^2}{2} |H_u \cdot H_d|^2.
 \end{aligned} \tag{28}$$

The spectrum consists of five physical Higgs bosons, two CP-even neutral, h^0 and H^0 , one CP-odd, A^0 and two charged, H^\pm . The Higgs potential depends only on three unknown parameters. One is fixed by $v^2 = 2(\langle H_u \rangle^2 + \langle H_d \rangle^2)$, while the other two can be traded by $\tan \beta$ and M_A . At tree level, the other Higgs masses are given by

$$\begin{aligned}
 M_{H^\pm}^2 &= M_A^2 + M_W^2, \\
 M_{h,H}^2 &= \frac{1}{2} \left\{ M_A^2 + M_Z^2 \mp \sqrt{(M_A^2 - M_Z^2)^2 + 4 \sin^2 2\beta M_A^2 M_Z^2} \right\},
 \end{aligned} \tag{29}$$

that leads to the prediction $M_h \leq M_Z$. Quantum corrections however change this prediction, making M_h very sensitive to the superparticle spectrum. In spite of this, the Higgs h^0 is always light, $M_h \lesssim 130$ GeV (see Fig. 10) and should be visible at the LHC through its decay to $b\bar{b}$, $\tau\tau$ or $\gamma\gamma$. The other Higgs bosons will also be visible at the LHC (Fig. 11) except in certain regions of $\tan \beta$ – M_A where they are difficult to be seen.

6.1 Superpartners at Hadron colliders

The hunting of superparticles at the Tevatron and the LHC is quite involved due to the large number of particles. In models with R -parity the superpartners are always produced in pairs and cascade down to the LSP that, being stable, goes away from the detectors. A typical example is gaugino hunting. Gauginos, once produced, can decay through different channels. For example, if $\tilde{\chi}^0$ is the LSP, we can have

$$\tilde{g} \rightarrow q\bar{q} \rightarrow q\bar{q}\tilde{\chi}^0. \tag{30}$$

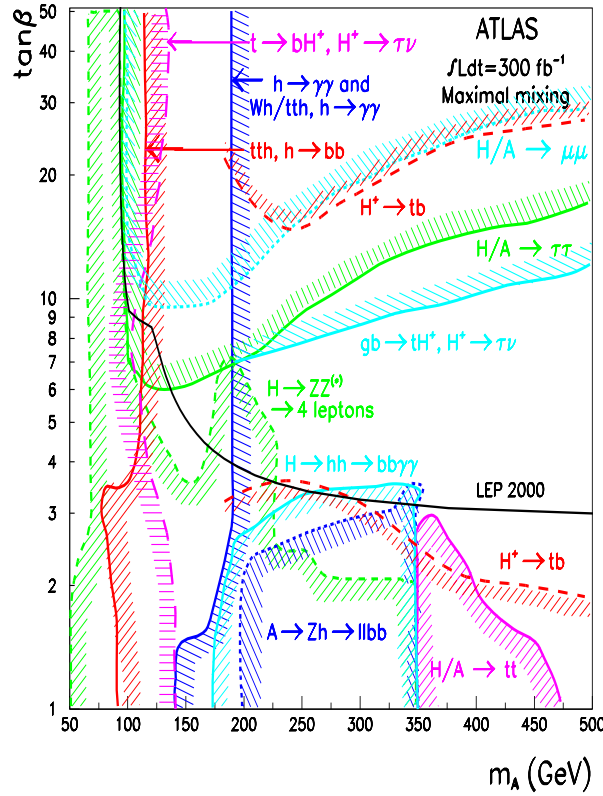


Fig. 11: Decay channels of the MSSM Higgs bosons that will be visible at the LHC as a function of $\tan \beta$ and M_A

In this case the signal consists of looking for an excess of jets+ missing E_T . In certain cases the decay can be

$$\tilde{g} \rightarrow q\tilde{q} \rightarrow qq\tilde{\chi}^+ \rightarrow qql^+\tilde{\nu} \rightarrow qql^+\nu\tilde{\chi}^0, \quad (31)$$

where $\tilde{\chi}^+$ is a chargino, a mixture of \tilde{W} and charged Higgsinos. The final signal has in this case two extra leptons that are easy to detect. For GMSB models, where the LSP is the gravitino, we have instead that $\tilde{\chi}^0$ decays to the gravitino, $\tilde{\chi}^0 \rightarrow \gamma + \tilde{G}$, and the final signal can be accompanied by photons. Searches on charginos require similar signatures, although the production cross-sections are obviously smaller.

In almost all cases, the Tevatron and, especially, the LHC can do a good job and reach super-particles up to very high masses. Figure 12 shows the expected sensitivity at the LHC for gaugino and neutralino searches for different luminosities. If supersymmetry is there, we have a good chance to discover it.

7 Higgsless and composite Higgs

Although the Higgs mechanism is a simple and economical way to break the electroweak gauge symmetry of the SM and at the same cure the bad high-energy behaviour of the $W_L W_L$ scattering amplitudes, it has, as we showed, an ‘expensive price to pay’: the hierarchy problem. For this reason, it is interesting to look for other ways to break the electroweak symmetry and unitarize the $W_L W_L$ scattering amplitudes. An example can be found in QCD, where pion-pion scattering is unitarized by the additional resonances that arise from the $SU(3)_c$ strong dynamics. A replica of QCD at energies \sim TeV that breaks the electroweak symmetry can then be an alternative to the Higgs mechanism. This is the so-called technicolor model [17] (TC). In TC there is no Higgs particle and the SM scattering amplitudes are unitarized, as in QCD, by infinite heavy resonances. One of the main obstacles to implementing this approach has arisen from EWPT that has disfavoured this type of model. The reason has been the following. Without

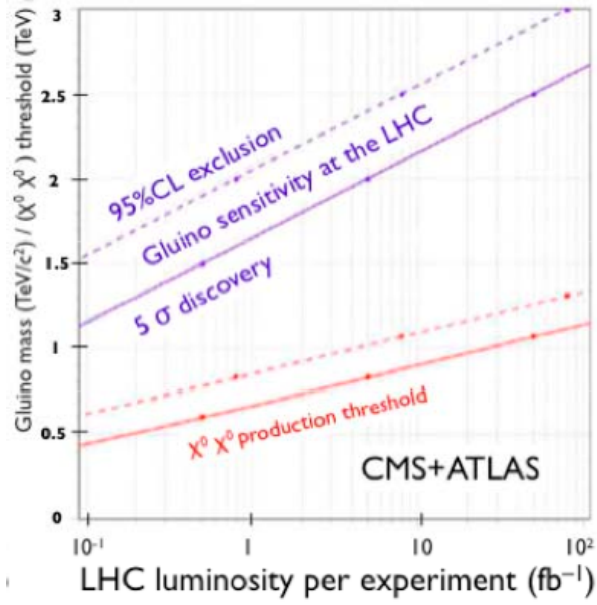


Fig. 12: Expected sensitivity at the LHC for \tilde{g} and $\tilde{\chi}^0$ searches for different luminosities

a Higgs, we expect the new particles responsible for unitarizing the SM amplitudes to have a mass at around 1 TeV. These same resonances give large tree-level contributions to the electroweak observables.

There have been two different alternatives to overcome this problem. Either one assumes that (1) there are extra contributions to the electroweak observables that make the model consistent with the experimental data, or (2) that the strong sector does not break the electroweak symmetry but it just delivers a composite pseudo-Goldstone boson (PGB) to be identified with the Higgs. This Higgs gets a potential at the one-loop level and triggers EWSB at lower energies.

In the first case, the Higgsless approach, EWPT are satisfied thanks to additional contributions to the electroweak observables that can come from extra scalars or fermions of the TC model, or from vertex corrections. As we will see, the cancellations needed to pass the EWPT are not large, making this possibility not so inconceivable.

In the second case, the Higgs plays the role of *partly* unitarizing the SM scattering amplitudes. Compared to theories without a Higgs, the scale at which new dynamics is needed can be delayed, and therefore the extra resonances that ultimately unitarize the SM amplitudes can be heavier. In this case EWPT will be under control. This is the approach of the composite Higgs models, first considered by Georgi and Kaplan [18]. In these theories a light Higgs arises as a PGB of a strongly interacting theory, in a very similar way to pions in QCD.

Although these scenarios offer an interesting completion of the SM, the difficulty of calculating within strongly coupled theories has been a deterrent from fully exploring them. Nevertheless, the situation has changed in recent years. Inspired by the AdS/CFT correspondence [19], a new approach to building realistic and predictive Higgsless and composite Higgs models has been developed. The AdS/CFT correspondence states that weakly coupled five-dimensional (5D) theories in Anti-de-Sitter (AdS) have a 4D holographic description in terms of strongly coupled conformal field theories (CFT). Such correspondence gives a definite prescription on how to construct five-dimensional theories that have the same physical behaviour and symmetries as the desired strongly coupled 4D theory. This has allowed one to propose concrete Higgsless [20] and composite Higgs [21, 22] models that not only are consistent with the experimental constraints, but also give clear predictions for the physics at the LHC. We will briefly discuss them in the section on extra dimensions.

7.1 The original Technicolor model. Achievements and pitfalls

Technicolor models [17] of EWSB consist of a new strong gauge sector, $SU(N)$ or $SO(N)$, that it is assumed to confine at a low-scale $\mu_{IR} \sim \text{TeV}$. In addition, the model contains (at least) two flavours of techni-quarks $T_L^{u,d}, T_R^{u,d}$ transforming in the fundamental representation of the strong group and as ordinary quarks under the electroweak group. As occurs in QCD, this implies that the strong sector has a global $G = SU(2)_L \times SU(2)_R \times U(1)_X$ symmetry under which $T_L^{u,d}$ transforms as a $(\mathbf{2}, \mathbf{1})_{1/6}$ and $T_R^{u,d}$ transforms as a $(\mathbf{1}, \mathbf{2})_{1/6}$ (the hypercharge is given by $Y = 2(T_3^R + X)$). Assuming that the TC quarks condensate, $\langle \bar{T}_L T_R \rangle \sim \mu_{IR}^3$, the global symmetry of the strong sector G is broken down to $H = SU(2)_V \times U(1)_X$. The electroweak symmetry is then broken giving masses to the corresponding SM gauge bosons. Fermion masses are assumed to arise from higher-dimensional operators such as $\bar{q}_L u_R \bar{T}_R T_L / M^2$ that can be induced from an extended heavy gauge sector (ETC). After the TC-quark condensation, SM fermions get masses $m_u \sim \mu_{IR}^3 / M^2$.

If the number of colours N of the TC group is large enough, the strong sector can be described by an infinite number of resonances [23]. The masses and couplings of the resonances depend on the model. Nevertheless, as in QCD, we can expect vector resonances transforming as a triplet of $SU(2)_V$; the TC-rho of mass $m_\rho \sim \mu_{IR}$. In order to see the implications of these resonances on the SM observables, it is useful to write the low-energy Lagrangian of the SM fields obtained after integrating out the strong sector (the equivalent of the QCD chiral Lagrangian). It is convenient to express this Lagrangian in a $SU(2)_L \times SU(2)_R \times U(1)_X$ -symmetric way. To do so, we promote the elementary SM fields to fill complete representations of $SU(2)_L \times SU(2)_R \times U(1)_X$. For the bosonic sector, this means to introduce extra non-dynamical vectors, i.e., spurions, to complete the corresponding adjoint representations W_μ^L , W_μ^R , and B_μ . Having the Goldstone multiplet U parametrizing the coset $SU(2)_L \times SU(2)_R / SU(2)_V$, the bosonic low-energy Lagrangian is given by

$$\mathcal{L}_{\text{eff}} = f^2 \left[\frac{1}{4} |D_\mu U|^2 + \frac{c_S}{m_\rho^2} \text{Tr}[W_{\mu\nu}^L U W^{\mu\nu R} U^\dagger] + \dots \right], \quad (32)$$

where $D_\mu U = \partial_\mu U + iW_\mu^L U - iU W_\mu^R$ and f is the analog of the pion decay constant that scales as $f \sim \sqrt{N}/(4\pi) \times m_\rho$ [23]. In Eq. (32) we have omitted terms of order $(DU)^4$ that do not contribute to the SM gauge boson self-energies, and terms of order $f^2 D^2/m_\rho^4$ that are subleading for physics at energies below m_ρ . The c_S is an order-one coefficient that in QCD takes the value $c_S = L_{10} m_\rho^2 / f^2 \simeq -0.4$. The mass of the SM W arises from the kinetic term of U that gives $M_W^2 = g^2 f^2 / 4$ from which we can deduce

$$f = v \simeq 246 \text{ GeV} \quad \text{and} \quad m_\rho \simeq 2 \sqrt{\frac{3}{N}} \text{ TeV}. \quad (33)$$

We also obtain Eq. (6) due to the $SU(2)_V$ symmetry that corresponds to a custodial symmetry.

7.1.1 Flavour-changing neutral current (FCNC) and the top mass

If the SM fermion masses arise from an ETC sector that generate the operators $\bar{q}_L^i u_R^j \bar{T}_R T_L / M^2$, this sector also will generate FCNCs of order $\bar{q}_L^i u_R^j \bar{q}_L^k u_R^l / M^2$ that are larger than experimentally allowed. Also the top mass is too large to be generated from a higher dimensional operator. Solutions to these problems have been proposed [24]. Nevertheless most of the solutions cannot successfully pass EWPT.

7.1.2 Electroweak precision tests

The most important corrections to the electroweak observables coming from TC-like models are universal corrections to the SM gauge boson self-energies, $\Pi_{ij}(p)$, and non-universal corrections to $Zb\bar{b}$, $\delta g_b / g_b$. The universal corrections to the SM gauge bosons can be parametrized by four quantities: $\hat{S}, \hat{T},$

W and Y [25]. The first two, the most relevant ones for TC models [26], are defined as

$$\hat{S} = g^2 \Pi'_{W_3 B}(0), \quad \hat{T} = \frac{g^2}{M_W^2} [\Pi_{W_3 W_3}(0) - \Pi_{W^+ W^-}(0)]. \quad (34)$$

Since \hat{T} is protected by the custodial symmetry, the Lagrangian Eq. (32) only generates \hat{S} . We have

$$\hat{S} = -g^2 c_S \frac{f^2}{m_\rho^2} \simeq 2.3 \cdot 10^{-3} \left(\frac{N}{3} \right), \quad (35)$$

where we have extracted the result from QCD. Extra contributions to \hat{S} , beyond those of the SM, are constrained by the experimental data. They must be smaller than⁹ $\hat{S} \lesssim 2 \cdot 10^{-3}$ at 99% CL. We see that the contribution Eq. (35) is at the edge of the allowed value. Models with N larger than 3 or with an extra generation of TC quarks, needed for realistic constructions (ETC models), are therefore ruled out. The bound $\hat{S} \lesssim 2 \cdot 10^{-3}$ can be saturated only if \hat{T} receives extra positive contributions $\sim 5 \cdot 10^{-3}$ beyond those of the SM. Although the custodial $SU(2)_V$ symmetry of the TC models guarantees the vanishing of the TC contributions to the \hat{T} -parameter, one-loop contributions involving both the top and the TC sector are nonzero. Nevertheless, in strongly interacting theories we cannot reliably calculate these contributions and know whether they give the right amount to \hat{T} .

As we said before, the generation of a top mass around the experimental value is difficult to achieve in TC models and requires new strong dynamics beyond the original sector [24]. Even if a large enough top mass is generated, an extra difficulty arises from $Zb\bar{b}$. On dimensional grounds, assuming that $t_{L,R}$ couples with equal strength to the TC sector responsible for EWSB, we have the estimate $\delta g_b/g_b \sim m_t/m_\rho \gtrsim 0.07$ that overwhelms the experimental bound $|\delta g_b/g_b| \lesssim 5 \cdot 10^{-3}$. Similar conclusions are reached even if $t_{L,R}$ couples with different strength to the TC sector [21], unless the custodial symmetry is preserved by the b_L coupling [27].

Realistic extra-dimensional Higgsless models can be constructed in which the above problems can be overcome, although this requires extra new assumptions and some adjustments of the parameters of the model [28].

7.2 Composite PGB Higgs

By enlarging the group G , while keeping qualitatively the same properties of the Higgsless models described above, we are driven to a different scenario in which the strong sector instead of breaking the electroweak symmetry, contains a light Higgs in its spectrum that will be the responsible for EWSB. The minimal model consists of a strong sector with the following symmetry breaking pattern [21]:

$$SO(5) \rightarrow SO(4). \quad (36)$$

It contains four Goldstone bosons parametrized by the $SO(5)/SO(4)$ coset:

$$\Sigma = \langle \Sigma \rangle e^{\Pi/f}, \quad \langle \Sigma \rangle = (0, 0, 0, 0, 1), \quad \Pi = \begin{pmatrix} 0_4 & h_a \\ -h_a^T & 0 \end{pmatrix}, \quad (37)$$

where h_a ($a = 1, \dots, 4$) is a real 4-component vector, which transforms as a doublet under $SU(2)_L \in SO(4)$. This is identified with the Higgs. Instead of following the TC idea for fermion masses described before, we can assume, inspired by extra dimensional models [21], that the SM fermion couples linearly to fermionic resonances of the strong sector. This can lead to correct fermion masses without severe FCNC problems.

⁹Since TC models do not have a Higgs, we are taking the result of Ref. [25] for $M_h \simeq 1$ TeV.

The low-energy theory for the PGB Higgs, written in a $SO(5)$ -invariant way, is given by

$$\mathcal{L}_{\text{eff}} = f^2 \left[\frac{1}{2} (D_\mu \Sigma) (D^\mu \Sigma)^T + \frac{c_S}{m_\rho^2} \Sigma F_{\mu\nu} F^{\mu\nu} \Sigma^T + V(\Sigma) + \dots \right], \quad (38)$$

where $F_{\mu\nu}$ is the field-strength of the $SO(5)$ gauge bosons (only the SM bosons must be considered dynamical). From the kinetic term of Σ we obtain $M_W^2 = g^2 (s_h f)^2 / 4$ together with Eq. (6), where we have defined $s_h \equiv \sin h/f$ with $h = \sqrt{h_a^2}$. This implies

$$v = s_h f \simeq 246 \text{ GeV}. \quad (39)$$

In this model the contribution to \hat{S} has an extra suppression factor v^2/f^2 as compared to Eq. (35), and then for $v \ll f$ one can satisfy the experimental constraint. Also $\delta g_b/g_b$ can be under control due to the custodial symmetry [27]. The exact value of v/f comes from minimizing the Higgs potential $V(h)$ that arises at the loop level from SM couplings to the strong sector that break the global $SO(5)$ symmetry. The dominant contribution comes at one-loop level from the elementary $SU(2)_L$ gauge bosons and top quark. In the model of Ref. [22], the potential is approximately given by

$$V(h) \simeq \alpha s_h^2 - \beta s_h^2 c_h^2, \quad (40)$$

where α and β are constants induced at the one-loop level. For $\alpha < \beta$ and $\beta \geq 0$ we have that the electroweak symmetry is broken and, if $\beta > |\alpha|$, the minimum of the potential is at

$$s_h = \sqrt{\frac{\beta - \alpha}{2\beta}}. \quad (41)$$

To have $s_h < 1$ as required, we need $\alpha \sim \beta$ that can be accomplished in certain regions of the parameter space of the models. The physical Higgs mass is given by

$$M_h^2 \simeq \frac{8\beta s_h^2 c_h^2}{f^2}. \quad (42)$$

Since β arises from one-loop effects, the Higgs is light. In the extra-dimensional composite Higgs models [22] one obtains $f \gtrsim 500 \text{ GeV}$, $m_\rho \gtrsim 2.5 \text{ TeV}$, and $M_h \sim 100\text{--}200 \text{ GeV}$.

In recent years similar ideas based on Higgs as a PGB have also been put forward under the name of Little Higgs (LH) models [29]. In these models, however, the gauge and fermion sector is extended in order to guarantee that Higgs mass corrections arise at the two-loop level instead of one-loop, allowing for a better insensitivity of the electroweak scale to the strong sector scale m_ρ .

7.3 LHC phenomenology

7.3.1 Heavy resonances at the LHC

The universal feature of strongly coupled theories of EWSB or their extra dimensional analogs is the presence of vector resonances, triplet under $SU(2)_V$, of masses in the range 0.5–2.5 TeV; they are the TC-rho or Kaluza–Klein states of the W_μ . They can either be produced in a $q\bar{q}$ Drell–Yan scattering or via weak boson fusion. These vector resonances will mostly decay into pairs of longitudinally polarized weak bosons (or, if possible, to a weak boson plus a Higgs), and to pairs of tops and bottoms. Studies at the LHC have been devoted to a very light TC-rho, $m_\rho \lesssim 600 \text{ GeV}$, that will be able to be seen for an integrated luminosity of 4 fb^{-1} [30].

In extra-dimensional Higgsless and composite Higgs models one also expects heavy gluon resonances. Their dominant production mechanism at the LHC is through $u\bar{u}$ or $d\bar{d}$ annihilation, decaying mostly in top pairs. The signal will then be a bump in the invariant $t\bar{t}$ mass distribution. For an integrated luminosity of 100 fb^{-1} the reach of the gluon resonances can be up to masses of 4 TeV [31].

The most promising way to unravel some composite Higgs model is by detecting heavy fermions with electric charge $5/3$ ($q_{5/3}^*$) [22]. For not-too-large values of its mass $m_{q_{5/3}^*}$, roughly below 1 TeV, these new particles will be mostly produced in pairs, via QCD interactions,

$$q\bar{q}, gg \rightarrow q_{5/3}^* \bar{q}_{5/3}^*, \quad (43)$$

with a cross-section completely determined in terms of $m_{q_{5/3}^*}$. Once produced, $q_{5/3}^*$ will mostly decay to a (longitudinally polarized) W^+ plus a top quark. The final state of the process Eq. (43) consists then mostly of four W 's and two b -jets:

$$q_{5/3}^* \bar{q}_{5/3}^* \rightarrow W^+ t W^- \bar{t} \rightarrow W^+ W^+ b W^- W^- \bar{b}. \quad (44)$$

Using same-sign dilepton final states we could discover these particles for masses of 500 GeV (1 TeV) for an integrated luminosity of 100 pb^{-1} (20 fb^{-1}) [32]. For increasing values of $m_{q_{5/3}^*}$ the cross-section for pair production quickly drops, and single production might become more important; masses up to 1.5 TeV could be reached at the LHC [33].

Besides $q_{5/3}^*$, certain composite models and LH models also predict states of electric charge $2/3$ or $-1/3$ that could also be produced in pairs via QCD interactions or singly via bW or tW fusion [34, 35]. They will decay to a SM top or bottom quark plus a longitudinally polarized W or Z , or a Higgs. When kinematically allowed, a heavier resonance will also decay to a lighter one accompanied with a W_{long} , Z_{long} or h . Decay chains could lead to extremely characteristic final states. For example, in one of the models of Ref. [22], the Kaluza–Klein with charge $2/3$ is predicted to be generally heavier than $q_{5/3}^*$. If pair produced, they can decay to $q_{5/3}^*$ leading to a spectacular six W 's plus two b -jets final state:

$$q_{2/3}^* \bar{q}_{2/3}^* \rightarrow W^- q_{5/3}^* W^+ \bar{q}_{5/3}^* \rightarrow W^- W^+ W^+ b W^+ W^- W^- \bar{b}. \quad (45)$$

In conclusion, our brief discussion shows that there are characteristic signatures predicted by these models that will distinguish them from other extensions of the SM. While certainly challenging, these signals will be extremely spectacular, and will provide an indication of a new strong dynamics responsible for EWSB.

7.3.2 Experimental tests of a composite Higgs

As an alternative to the detection of heavy resonances, the composite Higgs scenario can also be tested by measuring the couplings of the Higgs and seeing differences from those of a SM point-like Higgs. For small values of $\xi \equiv v^2/f^2$, as needed to satisfy the constraint on \hat{S} , we can expand the low-energy Lagrangian in powers of h/f and obtain in this way the following dimension-6 effective Lagrangian involving the Higgs doublet H :

$$\begin{aligned} \mathcal{L}_{\text{SILH}} = & \frac{c_H}{2f^2} \partial^\mu (H^\dagger H) \partial_\mu (H^\dagger H) + \frac{c_T}{2f^2} (H^\dagger \overleftrightarrow{D}^\mu H) (H^\dagger \overleftrightarrow{D}_\mu H) \\ & - \frac{c_6 \lambda}{f^2} (H^\dagger H)^3 + \left(\frac{c_y y f}{f^2} H^\dagger H \bar{\psi}_L H \psi_R + \text{h.c.} \right). \end{aligned} \quad (46)$$

Equation (46) will be referred to as the Strongly Interacting Light Higgs (SILH) Lagrangian [36]. We have neglected operators suppressed by $1/m_\rho^2$ that are subleading versus those of Eq. (46) by a factor $f^2/m_\rho^2 \sim N/(16\pi^2)$, or operators that do not respect the global symmetry G and therefore are only induced at the one-loop level with extra suppression factors — see Ref. [36]. The coefficients c_H , c_T , c_6 , and c_y are constants of order one that depend on the particular models. In 5D composite Higgs models they take, at tree-level, the value [36]: $c_H = 1$, $c_T = 0$, $c_y = 1$ (0), and $c_6 = 0$ (1) for the model of Ref. [22] ([21]). Only the coefficient c_T is highly constrained by the experimental data, since it contributes to the \hat{T} -parameter. Nevertheless all models with an approximate custodial symmetry give

a small contribution to c_T . The other operators can only be tested in Higgs physics. They modify the Higgs decay widths according to

$$\begin{aligned}
\Gamma(h \rightarrow f\bar{f})_{\text{SILH}} &= \Gamma(h \rightarrow f\bar{f})_{\text{SM}} [1 - \xi(2c_y + c_H)] \\
\Gamma(h \rightarrow WW)_{\text{SILH}} &= \Gamma(h \rightarrow WW^{(*)})_{\text{SM}} [1 - \xi c_H] \\
\Gamma(h \rightarrow ZZ)_{\text{SILH}} &= \Gamma(h \rightarrow ZZ^{(*)})_{\text{SM}} [1 - \xi c_H] \\
\Gamma(h \rightarrow gg)_{\text{SILH}} &= \Gamma(h \rightarrow gg)_{\text{SM}} [1 - \xi \text{Re}(2c_y + c_H)] \\
\Gamma(h \rightarrow \gamma\gamma)_{\text{SILH}} &= \Gamma(h \rightarrow \gamma\gamma)_{\text{SM}} \left[1 - \xi \text{Re} \left(\frac{2c_y + c_H}{1 + J_\gamma/I_\gamma} + \frac{c_H}{1 + I_\gamma/J_\gamma} \right) \right] \\
\Gamma(h \rightarrow \gamma Z)_{\text{SILH}} &= \Gamma(h \rightarrow \gamma Z)_{\text{SM}} \left[1 - \xi \text{Re} \left(\frac{2c_y + c_H}{1 + J_Z/I_Z} + \frac{c_H}{1 + I_Z/J_Z} \right) \right].
\end{aligned} \tag{47}$$

The loop functions I and J are given in Ref. [36]. Note that the contribution from c_H is universal for all Higgs couplings and therefore it does not affect the Higgs branching ratios, but only the total decay width and the production cross-section. The measure of the Higgs decay width at the LHC is very difficult and it can only be reasonably done for a rather heavy Higgs, well above the two gauge boson threshold, that is not the case of a composite Higgs. However, for a light Higgs, LHC experiments can measure the product $\sigma_h \times BR_h$ in many different channels: production through gluon, gauge-boson fusion, and top-strahlung; decay into b , τ , γ and (virtual) weak gauge bosons. In Fig. 13, we show the prediction of a 5D composite Higgs for the relative deviation from the SM expectation in the main channels for Higgs discovery at the LHC. At the LHC with about 300 fb^{-1} , it will be possible to measure Higgs production rate times branching ratio in the various channels with 20–40 % precision [37]. This will translate into a sensitivity on $|c_H \xi|$ and $|c_y \xi|$ up to 0.2–0.4, at the edge of the theoretical predictions. Since the Higgs coupling determinations at the LHC will be limited by statistics, they can benefit from a luminosity upgrade, like the SLHC. At a linear collider, like the ILC, precisions on $\sigma_h \times BR_h$ can reach the per cent level [38], providing a very sensitive probe on the scale f .

Deviations from the SM predictions of Higgs production and decay rate, could be a hint towards models with strong dynamics. Nevertheless, they do not unambiguously imply the existence of a new strong interaction. The most characteristic signals of the SILH Lagrangian have to be found in the very high-energy regime. Indeed, a peculiarity of the SILH Lagrangian is that, in spite of a light Higgs, longitudinal gauge-boson scattering amplitudes grow with energy and the corresponding interaction can become sizeable. Indeed, the extra Higgs kinetic term proportional to $c_H \xi$ in Eq. (46) prevents Higgs exchange diagrams from accomplishing the exact cancellation, present in the SM, of the terms growing with energy in the amplitudes. Therefore, although the Higgs is light, we obtain strong WW scattering at high energies. Using the equivalence theorem [39], it is easy to derive the following high-energy limit of the scattering amplitudes for longitudinal gauge bosons:

$$\begin{aligned}
\mathcal{A}(Z_L^0 Z_L^0 \rightarrow W_L^+ W_L^-) &= \mathcal{A}(W_L^+ W_L^- \rightarrow Z_L^0 Z_L^0) = -\mathcal{A}(W_L^\pm W_L^\pm \rightarrow W_L^\pm W_L^\pm) \\
\mathcal{A}(W_L^\pm W_L^\pm \rightarrow W_L^\pm W_L^\pm) &= -\frac{c_H s}{f^2},
\end{aligned} \tag{48}$$

$$\mathcal{A}(W^\pm Z_L^0 \rightarrow W^\pm Z_L^0) = \frac{c_H t}{f^2}, \quad \mathcal{A}(W_L^+ W_L^- \rightarrow W_L^+ W_L^-) = \frac{c_H(s+t)}{f^2}, \tag{49}$$

$$\mathcal{A}(Z_L^0 Z_L^0 \rightarrow Z_L^0 Z_L^0) = 0. \tag{50}$$

8 Extra dimensions

One of the first to postulate the existence of extra dimensions was Kaluza in 1921 [40]. He wanted to unify gravity with electromagnetism. For this purpose he considered a 5D theory with only gravity.

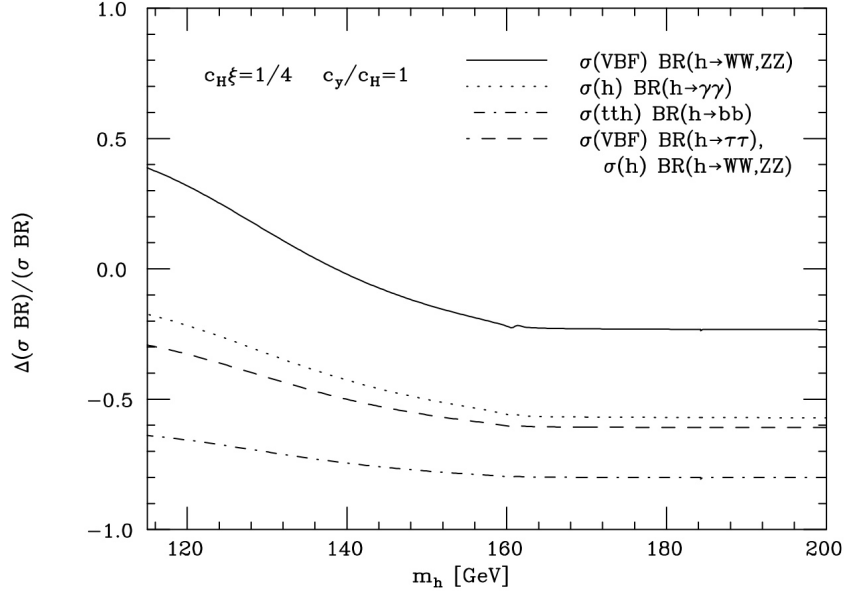


Fig. 13: The deviations from the SM predictions of Higgs production cross-sections (σ) and decay branching ratios (BR) defined as $\Delta(\sigma BR)/(\sigma BR) = (\sigma BR)_{\text{SILH}}/(\sigma BR)_{\text{SM}} - 1$. The predictions are shown for some of the main Higgs discovery channels at the LHC with production via vector-boson fusion (VBF), gluon fusion (h), and top-strahlung (tth).

The 5D gravitons, h_{MN} ($M, N = \mu, 5$), corresponds to fluctuations around the flat space¹⁰ $g_{MN} = \eta_{MN} + h_{MN}$. From a 4D point of view, they decompose as a spin-2 particle, $h_{\mu\nu}$, a spin-1, $h_{\mu 5}$, and spin-zero h_{55} . Kaluza associated the first one with the 4D graviton, and the second one with the photon. This was possible due to the fact that $h_{\mu 5}$ transforms, under an infinitesimal translation in the extra dimension $y \rightarrow y + \theta(x)$, as $h_{\mu 5} \rightarrow h_{\mu 5} - \partial_\mu \theta$ that corresponds to a 4D gauge transformation. Although this 5D theory gave a unified picture of gravity and electromagnetism, it failed to be realistic since it could not incorporate massless charged matter.

A second motivation to consider higher-dimensional theories came from string theory [41]. Strings were found to give a consistent description of quantum gravity. Nevertheless, this could only happen if strings were living in more than 4 dimensions. For example, superstring theory must be formulated in 10 dimensions. Therefore extra dimensions can be needed in order to have a consistent description of quantum gravity. As we will explain below, it was necessary for these extra dimensions to be compactified with a compactification radii around the Planck length $R \sim 10^{-32}$ cm, and therefore out of the reach of any experiment.

In 1998, however, Arkani-Hamed, Dimopoulos, and Dvali realized that extra dimensions could be larger than the Planck length if only gravity was propagating in these extra dimensions [42]. Furthermore, the existence of extra dimensions for gravity could also explain why gravity was much weaker than the other interactions. The basic idea is very simple. If gravity propagates in $4 + d$ dimensions we know, by Gauss's law, that the force between two bodies of masses m_1 and m_2 separated by a distance r is given by

$$F = G_{\text{grav}} \frac{m_1 m_2}{r^{2+d}}, \quad (51)$$

where G_{grav} is the equivalent to Newton's constant in $4 + d$ dimensions. From Eq. (51) one learns that the gravity force can be weaker than the other gauge forces, not because the strength of the interaction, G_{grav} , is small, but because gravity propagates in more than 4D and then the gravity force decreases

¹⁰Our convention here is $\eta_{MN} = \text{Diag}(-1, 1, 1, 1, 1)$.

faster as r increases, $F \sim 1/r^{2+d}$, than the gauge forces, $F \sim 1/r^2$. Of course, we know that at very large distances gravity lives in 4D, since we know that Newton's law reproduces very accurately, for example, the orbits of the planets. This means that the extra dimensions must be compact with a compactification radius R . At distances larger than R we will have a 4D theory with Newton's law:

$$F = G_N \frac{m_1 m_2}{r^2}, \quad (52)$$

where G_N is the observed Newton's constant. Matching Eqs. (51) and (52) at $r = R$ one gets

$$G_N = \frac{G_{grav}}{R^d}. \quad (53)$$

Therefore large R implies a small G_N . In other words, 4D gravity must be weaker than the other interactions if its field lines spread over large extra dimensions. The larger the extra dimensions, the weaker is gravity. This is a very interesting possibility that, as we will see below, has spectacular phenomenological implications.

Several years later Randall and Sundrum found a different reason to have extra dimensions [43]. If the extra dimensions were curved or 'warped', gravitons would behave differently than gauge bosons and this could explain their different couplings to matter.

Below we will discuss these two scenarios in more detail. Let us first explain the situation in the old Kaluza–Klein picture.

9 Kaluza–Klein theories

As we said before, Kaluza was one of the first to consider theories with more than four dimensions in an attempt to unify gravity with electromagnetism. Klein developed this idea in 1926 using a formalism that is usually called Kaluza–Klein reduction [44]. Although their initial motivation and ideas do not seem to be viable, the formalism that they and others developed is still useful nowadays. This is the one that will be considered below.

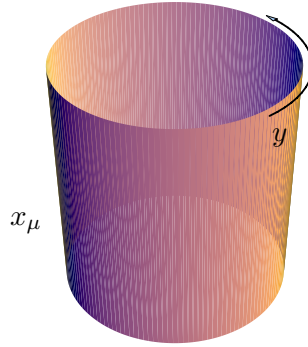


Fig. 14: Compactification on S^1

For simplicity, we will start with a 5D field theory of scalars. The action is given by

$$S_5 = - \int d^4x dy M_* \left[|\partial_\mu \phi|^2 + |\partial_y \phi|^2 + g_5^2 |\phi|^4 \right], \quad (54)$$

where by y we refer to the extra fifth dimension. We have extracted a universal scale M_* in front of the action in order to keep the 5D field with the same mass-dimension as in 4D. Let us now consider that the fifth dimension is compact and flat. We will consider that it has the topology of a circle S^1 as in Fig. 14.

This corresponds to the identification of y with $y + 2\pi R$. In such a case, we can expand the 5D complex scalar field in Fourier series:

$$\phi(x, y) = \sum_{n=-\infty}^{\infty} e^{iny/R} \phi^{(n)}(x) = \phi^{(0)}(x) + \sum_{n \neq 0} e^{iny/R} \phi^{(n)}(x), \quad (55)$$

that inserted in Eq. (54) and integrated over y gives

$$S_5 = S_4^{(0)} + S_4^{(n)} \quad (56)$$

where

$$S_4^{(0)} = - \int d^4x \, 2\pi R M_* \left[|\partial_\mu \phi^{(0)}|^2 + g_5^2 |\phi^{(0)}|^4 \right], \quad (57)$$

$$S_4^{(n)} = - \int d^4x \, 2\pi R M_* \sum_{n \neq 0} \left[|\partial_\mu \phi^{(n)}|^2 + \left(\frac{n}{R}\right)^2 |\phi^{(n)}|^2 \right] + \text{quartic} - \text{couplings}. \quad (58)$$

We see that the above action corresponds to a 4D theory with a massless scalar $\phi^{(0)}$ and a tower of massive modes $\phi^{(n)}$. The field $\phi^{(0)}$ will be referred to as the zero-mode, while $\phi^{(n)}$ will be referred to as Kaluza–Klein (KK) modes.

This reduction of a 5D theory to a 4D theory allows one to treat 5D theories as 4D field theories. This is very useful since we know much more about 4D theories than 5D theories. At low energies (large distances) we know that massive states in 4D theories can be neglected. Therefore the effective theory at energies below $1/R$ is described by the zero-mode Eq. (57). After normalizing $\phi^{(0)}$, we obtain from Eq. (57)

$$S_4^{(0)} = - \int d^4x \left[|\partial_\mu \phi^{(0)}|^2 + g_4^2 |\phi^{(0)}|^4 \right], \quad (59)$$

where the 4D self-coupling is given by

$$g_4^2 = \frac{g_5^2}{2\pi R M_*}. \quad (60)$$

This equation tells us that the strength of the interaction of the zero-mode decreases as the radius increases. If R is large, the scalar is weakly coupled.

The general features described above for a 5D scalar will also hold for gauge fields and gravity. After Kaluza–Klein reduction, we will have a 4D theory with a massless gauge field and a graviton:

$$\begin{aligned} & - \int d^4x \, dy \, M_* \left[\frac{M_*^2}{2} \mathcal{R} + \frac{1}{4g_5^2} F^{MN} F_{MN} \right] \\ &= - \int d^4x \, M_* \left[\pi R M_*^2 \mathcal{R}^{(0)} + \frac{2\pi R}{4g_5^2} F^{(0)\mu\nu} F_{\mu\nu}^{(0)} \right] + \dots \\ &\equiv - \int d^4x \left[\frac{1}{16\pi G_N} \mathcal{R}^{(0)} + \frac{1}{4g_4^2} F^{(0)\mu\nu} F_{\mu\nu}^{(0)} \right] + \dots, \end{aligned} \quad (61)$$

where $\mathcal{R}^{(0)}$ is the 4D scalar-curvature containing the zero-mode (massless) graviton, and $F^{(0)}$ is the gauge field-strength of the zero-mode (massless) gauge boson. From Eq. (61) we read

$$g_4^2 = \frac{g_5^2}{2\pi R M_*}, \quad (62)$$

for the 4D gauge coupling, and

$$G_N = \frac{1}{16\pi^2 R M_*^3}, \quad (63)$$

for the 4D Newton constant. Again, as in Eq. (60), the strength of the interaction is suppressed by the length of the extra dimension.

Let us now imagine that we live in 5D. From Eqs. (62) and (63) we learn the following. Since the gauge couplings $g_4^2 = \mathcal{O}(1)$ and $g_5^2 \lesssim 1$ (in order to have a perturbative theory) we have from Eq. (62) that

$$R \sim \frac{1}{M_*}. \quad (64)$$

On the other hand, using the relation $G_N \equiv 1/(8\pi M_P^2)$, where $M_P = 2.4 \times 10^{18}$ GeV is from now on the *reduced* Planck scale, we have from Eq. (63) that

$$M_P^2 = 2\pi R M_*^3. \quad (65)$$

Equations (64) and (65) imply

$$R \sim \frac{1}{M_P} = l_P \sim 10^{-32} \text{ cm}. \quad (66)$$

We have then reached the conclusion that if we live in 5D, the radius of the extra dimension must be of order the Planck length l_P ! This extra dimension will not be accessible to present or near-future experiments. This is the reason why experimentalists never paid attention to the existence of extra dimensions even though they were motivated theoretically a long time ago, *e.g.*, from string theory.

Let us finish this section with a comment on the scale M_* . Classically, we introduced this scale based on dimensional grounds. At the quantum level, however, this scale has a similar meaning as M_P in 4D gravity or $1/\sqrt{G_F}$ in Fermi theory. It represents the cutoff Λ of the 5D theory. We do not know how to quantize the 5D theory above M_* , since amplitudes such as $\phi\phi \rightarrow \phi\phi$ grow with the energy as $\sim E/M_*$.

10 Large extra dimensions for gravity

In 1998 Arkani-Hamed, Dimopoulos, and Dvali (ADD) proposed a different scenario for extra dimensions [42]. Motivated by the weakness of gravity, they considered that only gravity was propagating in the extra dimension. As we already saw, the effective 5D theory at distances larger than R is a theory of 4D gravity with a G_N being suppressed by the length of the extra dimension. Then the smallness of G_N can be considered a consequence of large extra dimensions. The key point to avoid the conclusion of Eq. (66) is that not all fields should share the same dimensions. In particular, gauge bosons should be localized in a 4D manifold.

In 1995 string theorists realized that superstrings in the strong-coupling limit contain new solitonic solutions [45]. These solutions received the name of D-branes and consisted in sub-manifolds of dimensions $D+1$ (less than 10) with gauge theories living on them. From string theory we therefore learn that there can be theories where gravitons and gauge bosons do not share the same number of dimensions, giving realizations of the scenario proposed by ADD [46].

Let us then assume that gravity lives in more dimensions than the SM particles (leptons, quarks, the Higgs and gauge bosons), and study the implications of this scenario. First of all, we must find out how large the extra dimensions must be in order to reproduce the right value of G_N . For d flat and compact extra dimensions, we have

$$-\int d^4x d^d y M_*^d \frac{M_*^2}{2} \mathcal{R} = -\int d^4x V^d M_*^d \frac{M_*^2}{2} \mathcal{R}^{(0)} + \dots, \quad (67)$$

where V^d is the volume of the extra dimensions. Hence we have

$$M_P^2 = V^d M_*^{2+d}. \quad (68)$$

For a toroidal compactification we have $V^d = (2\pi R)^d$. Following Ref. [42], we will absorb the factors 2π in M_* and rewrite Eq. (68) as¹¹

$$M_P^2 = (RM_*)^d M_*^2. \quad (69)$$

Note that Eq. (64) does not apply since gauge bosons do not live in 5D. Let us fix M_* slightly above the electroweak scale $M_* \sim \text{TeV}$ to avoid introducing a new scale (this is a nullification of the hierarchy problem). In such a case we have from Eq. (69) a prediction for R :

$$\begin{aligned} d = 1 & \rightarrow R \sim 10^9 \text{ km}, \\ d = 2 & \rightarrow R \sim 0.5 \text{ mm}, \\ & \vdots \\ d = 6 & \rightarrow R \sim 1/(8 \text{ MeV}), \end{aligned}$$

The option $d = 1$ is clearly ruled out. For $d = 2$ we expect changes in Newton's law at distances below the mm. Surprisingly, as we will show below, we have not measured gravity at distances below ~ 0.1 mm. This is due to the fact that Van der Waals forces become comparable to gravity at distances around 1 mm, making it very difficult to disentangle gravity effects from the large Van der Waals effects. So the option $d = 2$ is being tested today at the present experiments. Larger values of d are definitely allowed.

10.1 Phenomenological implications

What are the implications of this scenario? Let us concentrate on the case $d = 2$. At distances shorter than 1 mm, we must notice that gravity lives in 6D. To study the effects of a 6D gravity, we will again Fourier decompose the 6D graviton field, $h_{\mu\nu}(x, y_1, y_2)$. For example, if y_1 and y_2 are compactified in a torus, we have the Fourier decomposition

$$h_{\mu\nu}(x, y_1, y_2) = \sum_{n_1=-\infty}^{\infty} \sum_{n_2=-\infty}^{\infty} e^{i(n_1 y_1 + n_2 y_2)/R} h_{\mu\nu}^{(\vec{n})}(x), \quad (70)$$

where $\vec{n} = (n_1, n_2)$. The state $h_{\mu\nu}^{(\vec{0})}$ is our massless graviton, while $h_{\mu\nu}^{(\vec{n})}$ with $\vec{n} \neq 0$ are the KK states that, from a 4D point of view, are massive particles of masses $m_{\vec{n}}^2 = (n_1^2 + n_2^2)/R^2$. Therefore we can describe this 6D theory as a 4D theory containing a massless graviton and a KK tower of graviton states. There are also the KK states for the components $h_{\mu 5}$, $h_{\mu 6}$, h_{65} , h_{55} , and h_{66} . Nevertheless, since matter is assumed to be confined in a 4D manifold at $y = 0$, we have that the energy-momentum tensor has only 4D components, $T_{MN} = \eta_M^\mu \eta_N^\nu T_{\mu\nu} \delta(y)$. Hence these extra states do not couple to the energy-momentum tensor of matter. The situation is a little bit more subtle for the 'dilaton' field ϕ that corresponds to a combination of h_{MN} $M, N = 5, 6$. Although it does not couple to $T_{\mu\nu}$, it mixes with the graviton. This mixing can be eliminated by a Weyl transformation. Nevertheless, after the Weyl rotation, ϕ appears to be coupled to the trace of $T_{\mu\nu}$. This coupling is usually smaller than those between gravitons and matter (in fact, it is zero for conformal theories) and therefore we will neglect it.

The effective Lagrangian for the KK gravitons, after normalizing the kinetic term of the gravitons is given by

$$\mathcal{L}_{KK} = \sum_{\vec{n} \neq 0} \left[\mathcal{L}_{kin} - \frac{1}{2} m_{\vec{n}}^2 \left(h^{(-\vec{n})\mu\nu} h_{\mu\nu}^{(\vec{n})} - h^{(-\vec{n})\mu}{}_{\mu} h^{(\vec{n})\nu}{}_{\nu} \right) + \frac{1}{M_P} h^{(\vec{n})\mu\nu} T_{\mu\nu} \right], \quad (71)$$

where \mathcal{L}_{kin} is the kinetic term of the gravitons. The KK states $h_{\mu\nu}^{(\vec{n})}$ will modify the gravitational interaction at $E > 1/R$. Since they couple to matter with a strength $\sim 1/M_P$, we have that at energies

¹¹ In string theory, where M_{st} plays the role of M_* , we have for $d = 6$ that $M_P^2 = 2\pi(RM_{st})^6 M_{st}^2/g_4^4$.

$E > 1/R$, the (dimensionless) gravitational strength squared grows as

$$g_{grav}^2 \sim \sum_{n_1=0}^{ER} \sum_{n_2=0}^{ER} \frac{E^2}{M_P^2} \sim (ER)^2 \frac{E^2}{M_P^2} \sim \left(\frac{E}{M_*} \right)^4, \quad (72)$$

where in the last equality we have used Eq. (69). Note that g_{grav}^2 becomes $\mathcal{O}(1)$ at energies M_* . Therefore M_* is the scale at which quantum gravity effects are important. The generalization to d extra dimensions is given by

$$g_{grav}^2 \sim \left(\frac{E}{M_*} \right)^{2+d}. \quad (73)$$

With Eq. (72) we can easily estimate any gravitational effect in any experimental process that we can imagine.

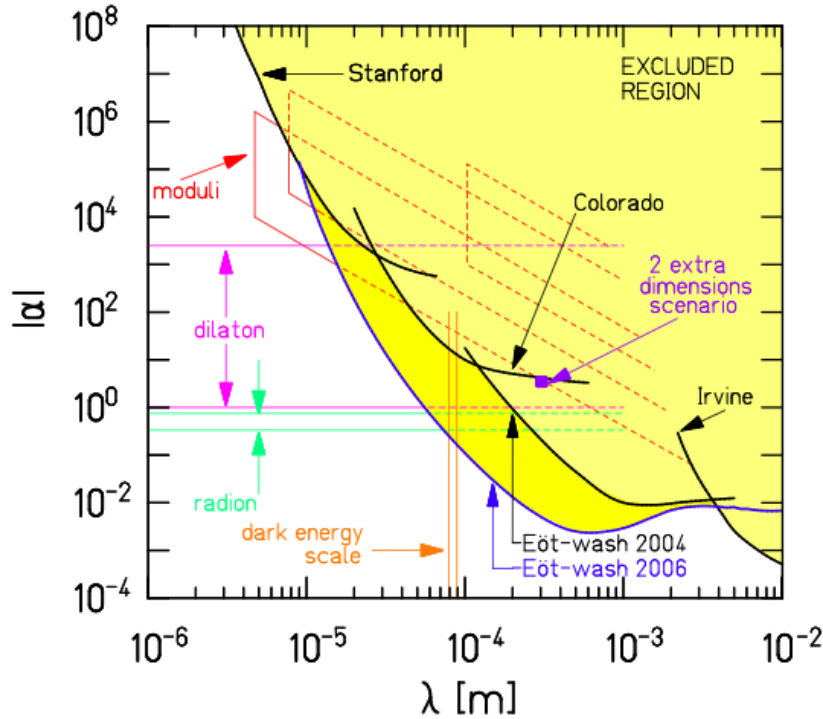


Fig. 15: Upper limits on forces of the form of Eq. (74) [47]

10.1.1 Measuring the gravitational force at millimetre distances

The KK of the graviton give rise to new forces. Since they are massive particles they produce a Yukawa-type force. For the first KK ($n_1 = \pm 1, n_2 = 0$ and $n_1 = 0, n_2 = \pm 1$) of masses $1/R$, this force is given by

$$F_{KK}(r) = -\alpha G_N \frac{m_1 m_2}{r} e^{-r/\lambda}, \quad (74)$$

where $\alpha = 16/3$ for a 2-torus compactification and $\lambda = R$. Searches for new forces have been carried out at several experiments. Nevertheless, the bounds on α are very weak at distances r below ~ 0.1 mm. In Fig. 15 we plot the present experimental bounds on α and λ . The value of $R \sim 0.5$ mm, expected for $M_* \sim \text{TeV}$ and $d = 2$, is ruled out. Therefore the case $d = 2$ is only at present allowed if $M_* \gtrsim 3$ TeV [47].

10.1.2 Collider experiments

It is easy to estimate that the contributions of the KK gravitons to any physical process measured in any collider are small. For example, let us consider the process $BR(K \rightarrow \pi + \text{gravitons})$. We can use Eq. (72) with $E \sim M_K$ and obtain

$$BR(K \rightarrow \pi + \text{gravitons}) \sim \left(\frac{M_K}{M_*} \right)^4 \sim 10^{-12}, \quad (75)$$

for $M_* \sim \text{TeV}$. This is close to the experimental constraint but it does not rule out the model. Similarly, we can estimate the contribution of the KK tower of gravitons to other low-energy processes:

$$\begin{aligned} BR(J/\Psi \rightarrow \gamma + \text{gravitons}) &\sim \left(\frac{M_{J/\Psi}}{M_*} \right)^4 \sim 10^{-10}, \\ BR(Z \rightarrow f\bar{f} + \text{gravitons}) &\sim \left(\frac{M_Z}{M_*} \right)^4 \sim 10^{-4}. \end{aligned} \quad (76)$$

None of them contradict the experimental bounds. Until now, no collider experiment has been able to exclude this scenario. The present limit from colliders arises from the process [48]

$$qq, gg \rightarrow g + \text{gravitons}, \quad (77)$$

where the gravitons disappear from the detector carrying energy with them. This cross-section grows with the energy as E^2/M_*^4 . Searching for a monojet plus missing transverse energy one can put a bound on M_* . From Tevatron, one gets $M_* \gtrsim 1 \text{ TeV}$ [49].

10.1.3 Astrophysics

Will this scenario modify stellar dynamics? The KK gravitons can be copiously produced in the stars. Since they interact very weakly with matter (with $1/M_P$ suppressed couplings) they can escape carrying energy with them. This can definitely change the stellar dynamics.

The most severe constraint on M_* arises from SN 1987A since it has a high core temperature $\sim 30 \text{ MeV}$. During the collapse of the SN 1987A about 10^{53} erg were released in a few seconds. We must then ensure that the graviton luminosity does not exceed 10^{53} erg/s .

Gravitons can be produced in the supernova core through several processes. One example is through nucleon scattering $NN \rightarrow NN + \text{Grav}$. This cross-section can be estimated to be $\sigma \sim \sigma(NN \rightarrow NN)(E/M_*)^{2+d}$. A detailed analysis leads to the bound $M_* \gtrsim 40, 3, 1 \text{ TeV}$ for $d = 2, 3, 4$ [50]. A more stringent bound can be found from KK gravitons emitted by supernova remnants and neutron stars that are gravitationally trapped, forming a halo, and occasionally decaying into photons. Limits on γ -rays from neutron-star sources imply [51] $M_* \gtrsim 200, 16 \text{ TeV}$ for $d = 2, 3$. The decay products of the KK gravitons that form the halo can provide an extra heat source if they hit the surface of the neutron star; the low measured luminosities of pulsars lead to $M_* \gtrsim 750, 35 \text{ TeV}$ for $d = 2, 3$. Although these bounds tell us that M_* must be larger than the electroweak scale, we must say that they are very sensitive to the masses of the lightest KK states that strongly depend on the type of compactification.

10.2 Future experiments

a) Gravity tests at sub-millimetre distances

As explained in Ref. [47], it is difficult to predict how much future experiments will improve on the tests of gravity at short-distances, because the results will almost surely be limited by systematic errors. We recall that only for the case $d = 2$ does one expect deviations from Newtonian gravity at accessible distances.

b) High-energy colliders

The graviton production Eq. (77) gives a very clean signature at the LHC: monojet+Missing energy. For the LHC with 10/fb, one expects to probe the model for a M_* up to ~ 8 TeV.

Another interesting signature of this scenario is the production of black holes [52]. In this scenario the Schwarzschild radius is of order

$$R_S \sim \left(\frac{M_{BH}}{M_*} \right)^{\frac{1}{1+d}} \frac{1}{M_*}, \quad (78)$$

where M_{BH} is the black hole mass (this is valid only for $M_{BH} > M_*$). Estimating the cross-section for the production of black holes as $\sigma \sim \pi R_S^2$, we will have for $M_* \sim \text{TeV}$ a production of 10^7 black holes at the LHC with a luminosity of 30 fb^{-1} .

11 Warped extra dimensions

There is another way to escape from the prediction of Eq. (66) that does not need to have the SM localized on a 4-dimensional boundary. This is based on having the extra dimension not flat but ‘warped’. This was realized by Randall and Sundrum (RS) [43]. Here we will describe this scenario and will study its phenomenological consequences. Again, as in the ADD scenario, the motivation is to explain why gravity is so weak.

The RS scenario is based on a 5D theory with the extra dimension y compactified in a orbifold, S^1/Z_2 . This compactification corresponds to a circle S^1 with the extra identification of y with $-y$ as shown in Fig. 16. This gives a ‘segment’ $y \in [0, \pi R]$, a manifold with boundaries¹² at $y = 0$ and

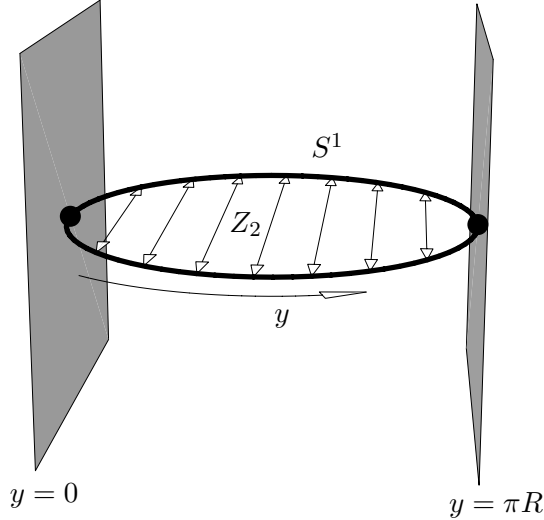


Fig. 16: The S^1/Z_2 orbifold

$y = \pi R$. Let us now assume that this 5D theory has a cosmological constant in the bulk and on the boundaries:

$$S_{RS} = - \int d^4x dy \sqrt{-g} \left[\frac{1}{2} M_*^3 \mathcal{R} + \Lambda + \delta(y) \Lambda_0 + \delta(y - \pi R) \Lambda_{\pi R} \right]. \quad (79)$$

By solving Einstein’s equations

$$\mathcal{R}_{MN} - \frac{1}{2} g_{MN} \mathcal{R} = - \frac{1}{M_*^3} T_{MN}, \quad (80)$$

¹² This is not a smooth manifold but seems to be a consistent compactification in string theory.

where

$$-T_{MN} = \Lambda g_{MN} + \Lambda_0 \delta(y) g_{\mu\nu} \delta_M^\mu \delta_N^\nu + \Lambda_{\pi R} \delta(y - \pi R) g_{\mu\nu} \delta_M^\mu \delta_N^\nu, \quad (81)$$

one obtains the metric

$$ds^2 = e^{2k|y|} dx^\mu dx^\nu \eta_{\mu\nu} + dy^2, \quad (82)$$

where

$$k = \sqrt{-\frac{\Lambda}{6M_*^3}}. \quad (83)$$

One must also impose $\Lambda_0 = -\Lambda_{\pi R} = \Lambda/k$. The metric Eq. (82) corresponds to a 5D Anti-de-Sitter (AdS) space (Fig. 17). The factor $e^{2k|y|}$ in front of dx^2 is called the ‘warp’ factor and determines how the 4D scale changes as we move inside the extra dimension. Since $\langle \mathcal{R} \rangle \propto k^2$, we must have $k \lesssim M_*$ in order to be able to use classical gravity.

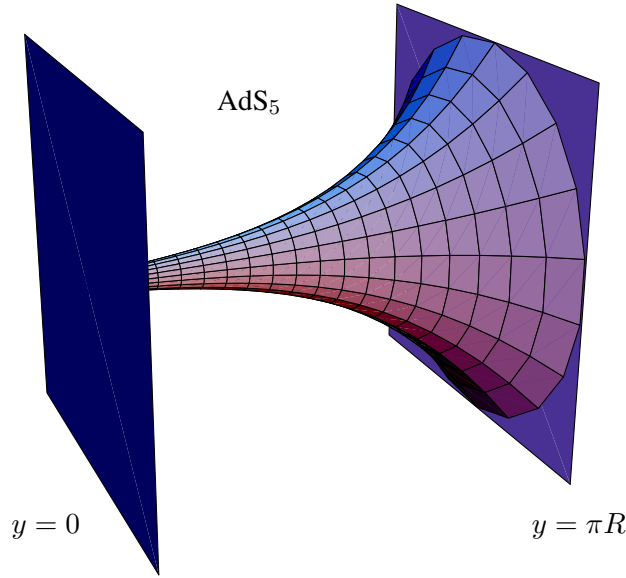


Fig. 17: The Randall–Sundrum scenario

Let us get the effective 4D theory at large distances (below the KK masses). We have

$$S_{RS} = - \int d^4x dy \sqrt{-g} \frac{1}{2} M_*^3 \mathcal{R} = - \int d^4x \int_0^{\pi R} dy e^{2ky} \frac{1}{2} M_*^3 \sqrt{-g^{(0)}} \mathcal{R}^{(0)} + \dots, \quad (84)$$

where we have used Eq. (82) with $\eta_{\mu\nu} \rightarrow \eta_{\mu\nu} + h_{\mu\nu}^{(0)}$ (the field $h_{\mu\nu}^{(0)}$ is the massless gravitational fluctuation about the classical solution Eq. (82)). Equation (84) implies that the Planck scale is given by

$$M_P^2 = \int_0^{\pi R} dy e^{2ky} M_*^3 = \frac{M_*^3}{2k} (e^{2k\pi R} - 1). \quad (85)$$

We see that the effect of having a warped space is that the relation between M_P , M_* , and R has changed from the flat case [Eq. (69)]. Equation (85) is telling us that M_P is exponentially larger than M_* . If M_* and k are fixed to be around the TeV, we can have $M_P \simeq 10^{18}$ GeV for an extra dimension of radius

$$R \simeq \frac{12}{k}. \quad (86)$$

Hence we generated a large M_P from a small (length of order $1/M_*$) extra dimension.

What about the gauge field [53]? From the 5D action

$$S_5 = - \int d^4x dy \sqrt{-g} \frac{M_*}{4g_5^2} F^{MN} F_{MN} , \quad (87)$$

we obtain the effective 4D theory of the zero-mode gauge field given by

$$S_5^{(0)} = - \int d^4x \int_0^{\pi R} dy \sqrt{-g^{(0)}} \frac{M_*}{4g_5^2} F^{(0)\mu\nu} F_{\mu\nu}^{(0)} . \quad (88)$$

Note that no exponential factors appear in front of the F^2 term. Equation (88) tells us that the 4D gauge coupling is given by

$$\frac{1}{g_4^2} = \int_0^{\pi R} dy \frac{M_*}{g_5^2} = \frac{\pi R M_*}{g_5^2} . \quad (89)$$

This is a very interesting result. It says that $1/g_4^2$ is not exponentially enhanced as in the case of the graviton. Therefore the gauge coupling can still be of order 1 and the model can be phenomenological viable without the need of localizing the gauge boson on the 4D boundary. We see that the warp factor of the metric Eq. (82) affects fields of different spin in a different way.

There is an alternative way to understand the features of the RS scenario. If we look at Eq. (82) we see that the 4D metric $g_{\mu\nu} = e^{2k|y|} \eta_{\mu\nu}$ changes as we move in the extra dimension. This means that the 4D scales are different in different points of the extra dimension. As a consequence, for an observer at $y = 0$, an experiment delivering an energy E at $y = 0$ will deliver an energy $E e^{ky}$ if the experiment is at y . The two energies are related by a blue-shift factor that is the square-root of the warp factor. From the point of view of effective theories this means that the cutoff of our theory depends on y . At $y = 0$ this is M_* , but at $y = \pi R$ this is $M_* e^{k\pi R}$. If we associate M_* with the electroweak scale, $M_* \sim \text{TeV}$, the electroweak symmetry breaking must take place at $y = 0$ (the Higgs must live at $y = 0$). The 4D graviton, however, must be living at $y = \pi R$ in order to have the Planck scale blue-shifted with respect to M_* , $M_P \sim M_* e^{k\pi R}$. This is exactly the situation of the RS scenario (as we will see below, the massless graviton is localized at $y = \pi R$) giving an intuitive explanation of Eq. (85).

11.1 KK reduction and phenomenology

In order to study the full implications of warped extra-dimensions one must study the effects of the KK states of the graviton, gauge fields, and fermions. Here we will perform a KK reduction of the graviton in the background (82).

First let us decompose the graviton as [54] $h_{\mu\nu}(x, y) = \sum_n f_n(y) h_{\mu\nu}^{(n)}(x)$. Since the metric Eq. (82) does not depend on x , $h_{\mu\nu}^{(n)}(x)$ corresponds to plane-waves $h_{\mu\nu}^{(n)}(x) \propto e^{ip_n x}$ where $p_n^2 = m_n^2$. From the linearized Einstein equation in the background (82) we obtain¹³

$$\left[\partial_y^2 - 4k^2 + e^{-2k|y|} m_n^2 - 4k[\delta(y) - \delta(y - \pi R)] \right] f_n(y) = 0 . \quad (90)$$

Solving this equation will give us the wave-functions f_n and the masses m_n . We obtain¹⁴

$$f_n(y) = \frac{1}{N_n} \left[J_2\left(\frac{m_n}{k} e^{-k|y|}\right) + b(m_n) Y_2\left(\frac{m_n}{k} e^{-k|y|}\right) \right] , \quad (91)$$

where J_α and Y_α are Bessel functions and N_n are normalization constants. The values of $b(m_n)$ and m_n are determined by the boundary conditions that give two equations

$$b(m_n) = - \frac{J_1\left(\frac{m_n}{k}\right)}{Y_1\left(\frac{m_n}{k}\right)} , \quad (92)$$

¹³We take the gauge $h_\mu^\mu = \partial_\mu h_\nu^\mu = 0$.

¹⁴We must also impose $f_n(y) = f_n(-y)$ due to the orbifold condition.

$$b(m_n) = b(m_n e^{-\pi k R}). \quad (93)$$

The values of m_n are therefore quantized. The lowest mode ($n = 0$) corresponds to a massless state $m_0 = 0$ (that we associate to the 4D graviton). It has a wave-function given by

$$f_0(y) = \frac{e^{2k|y|}}{N_0}. \quad (94)$$

We see that, as expected, this graviton is localized towards the $y = \pi R$ boundary. The RS scenario corresponds to a theory with localized gravity. The other KK have masses

$$m_n \simeq \left(n + \frac{1}{4}\right) \pi k. \quad (95)$$

These are of order of $k \sim \text{TeV}$. This is very different from the ADD scenario where the KK gravitons are very light. The wave-functions of the KK gravitons, f_n , are peaked towards the boundary at $y = 0$. Then they correspond to states localized at $y = 0$.

The phenomenology of the warped extra-dimensional scenario is very different from that of ADD. Since the KK are heavy, there is no implications for low-energy processes. Only accelerators at very high energies such as the LHC will be able to test this scenario. The process will be the same as for the ADD scenario, $qq, gg \rightarrow g + \text{gravitons}$ but now only a single KK state will be produced.

If the SM fields propagate in the extra dimension [55] (only the Higgs must live on the $y = 0$ boundary since, as we said above, the electroweak-breaking sector must be localized at $y = 0$), KK modes associated to the SM fields could be seen in future colliders.

11.2 The AdS/CFT correspondence, Higgsless and composite Higgs models

The AdS/CFT correspondence relates 5D theories of gravity in AdS to 4D strongly-coupled conformal field theories [19]. In the case of a slice of AdS (Fig. 17), a similar correspondence can also be formulated [56]. The boundary at $y = \pi R$ corresponds to an ultraviolet cutoff in the 4D CFT and to the gauging of certain global symmetries. For example, in the case we are considering where gravity and the SM gauge bosons live in the bulk, the corresponding 4D CFT will have the Poincaré group gauged (giving rise to gravity) and also the SM group $SU(3) \times SU(2)_L \times U(1)_Y$ (giving rise to the SM gauge bosons). Matter localized on the boundary at $y = \pi R$ corresponds to elementary fields external to the CFT that only interact via gravity and gauge interactions. On the other hand, the boundary at $y = 0$ corresponds in the dual theory to an infrared cutoff of the CFT. In other words, it corresponds to breaking the conformal symmetry at the TeV scale. The KK states of the 5D theory correspond to the bound states of the strongly coupled CFT. Although the CFT picture is useful for understanding some qualitative aspects of the theory, it is practically useless for obtaining quantitative predictions since the theory is strongly coupled. In this sense, the 5D gravitational theory in a slice of AdS represents a very useful tool since it allows one to calculate the particle spectrum, which would otherwise be unknown from the CFT side.

Following the AdS/CFT correspondence we can design five-dimensional models with the properties of the strongly-coupled models discussed in Section 7. For example, Higgsless models [20] consist in gauge theories in RS spaces with the symmetry pattern

$$\begin{array}{ll} \text{Boundary at } y = 0: & SU(2)_V \times U(1)_X \times SU(3)_c \\ \text{5D Bulk:} & SU(2)_L \times SU(2)_R \times U(1)_X \times SU(3)_c \\ \text{Boundary at } y = \pi R: & SU(2)_L \times U(1)_Y \times SU(3)_c \end{array} \quad (96)$$

For composite PGB Higgs models we have [21, 22]

$$\begin{array}{ll} \text{Boundary at } y = 0: & O(4) \times U(1)_X \times SU(3)_c \\ \text{5D Bulk:} & SO(5) \times U(1)_X \times SU(3)_c \\ \text{Boundary at } y = \pi R: & SU(2)_L \times U(1)_Y \times SU(3)_c \end{array} \quad (97)$$

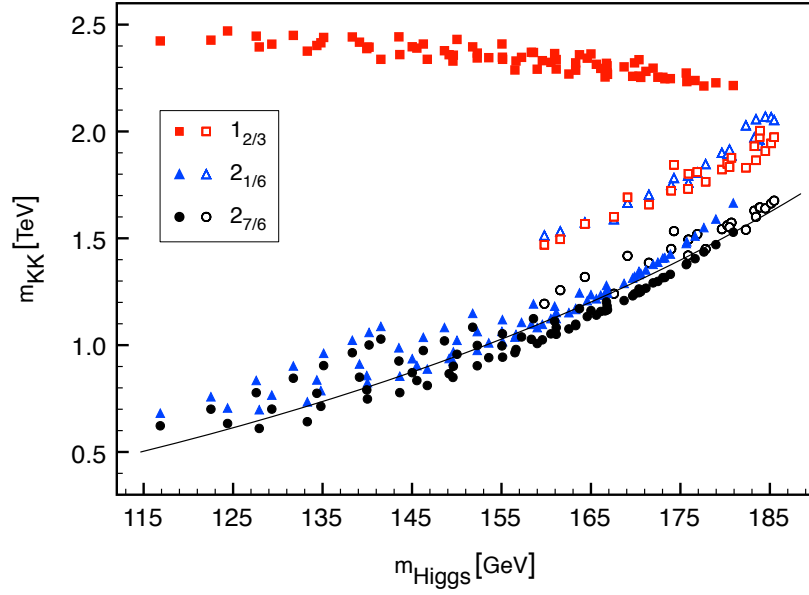


Fig. 18: KK fermion masses vs the Higgs mass in the model of Ref. [22]. All fermion KK states are colour triplets under the strong group. The quantum numbers under $SU(2)_L \times U(1)_Y$ are also given. We notice that the normalization of hypercharge in Ref. [22] is different from ours; one must multiply by 2 to get the hypercharges as defined here.

In these models the lightest KK states are the partners of the top with SM quantum numbers $(\mathbf{3}, \mathbf{2})_{7/3, 1/3}$ and $(\mathbf{3}, \mathbf{1})_{4/3}$. The spectrum is shown in Fig. 18. Gauge boson and graviton KK states are heavier, around 2.5 TeV and 4 TeV respectively.

Acknowledgements

I would like to thank the organizers of the 2010 European School of High-Energy Physics for such a successful and stimulating school. This work is partly supported by CICYT-FEDER-FPA2008-01430, 2009SGR894, UniverseNet (MRTN-CT-2006-035863), and ICREA Academia program.

References

- [1] K. Nakamura *et al.* [Particle Data Group Collaboration], J. Phys. G **G37** (2010) 075021.
- [2] See, for example, <http://project-gfitter.web.cern.ch/project-gfitter/>.
- [3] P. Sikivie, L. Susskind, M. B. Voloshin *et al.*, Nucl. Phys. **B173** (1980) 189.
- [4] J. Ellis, J. R. Espinosa, G. F. Giudice *et al.*, Phys. Lett. **B679** (2009) 369–375.
- [5] For a review on GUT, see, for example, P. Langacker, Phys. Rep. **72** (1981) 185; S. Raby, arXiv:hep-ph/0608183.
- [6] J. C. Pati and A. Salam, Phys. Rev. **D10** (1974) 275–289.
- [7] H. Georgi and S. L. Glashow, Phys. Rev. Lett. **32** (1974) 438–441.
- [8] P. Langacker and N. Polonsky, Phys. Rev. **D47** (1993) 4028–4045.
- [9] R. D. Peccei and H. R. Quinn, Phys. Rev. Lett. **38** (1977) 1440–1443.
- [10] S. Weinberg, Phys. Rev. Lett. **40** (1978) 223–226; F. Wilczek, Phys. Rev. Lett. **40** (1978) 279–282.
- [11] R. Haag, J. T. Lopuszanski, and M. Sohnius, Nucl. Phys. **B88** (1975) 257.
- [12] For a review, see, for example, S. P. Martin, in Kane, G. L. (ed.): Perspectives on supersymmetry 1–98. [hep-ph/9709356].

- [13] For a review, see, for example G. F. Giudice and R. Rattazzi, Phys. Rep. **322** (1999) 419–499.
- [14] G. R. Dvali, G. F. Giudice, and A. Pomarol, Nucl. Phys. **B478** (1996) 31–45.
- [15] A. H. Chamseddine, R. L. Arnowitt, and P. Nath, Phys. Rev. Lett. **49** (1982) 970; R. Barbieri, S. Ferrara, and C. A. Savoy, Phys. Lett. **B119** (1982) 343.
- [16] A. Djouadi, Phys. Rep. **459** (2008) 1–241.
- [17] S. Weinberg, Phys. Rev. D **13** (1976) 974; Phys. Rev. D **19** (1979) 1277; L. Susskind, Phys. Rev. D **20** (1979) 2619.
- [18] D. B. Kaplan and H. Georgi, Phys. Lett. B **136** (1984) 183; H. Georgi and D. B. Kaplan, Phys. Lett. B **145** (1984) 216.
- [19] J. M. Maldacena, Adv. Theor. Math. Phys. **2** (1998) 231; S. S. Gubser, I. R. Klebanov, and A. M. Polyakov, Phys. Lett. B **428** (1998) 105; E. Witten, Adv. Theor. Math. Phys. **2** (1998) 253.
- [20] C. Csaki, C. Grojean, L. Pilo, and J. Terning, Phys. Rev. Lett. **92** (2004) 101802; G. Burdman and Y. Nomura, Phys. Rev. D **69**, 115013 (2004); R. Barbieri, A. Pomarol, and R. Rattazzi, Phys. Lett. B **591** (2004) 141.
- [21] K. Agashe, R. Contino, and A. Pomarol, Nucl. Phys. B **719** (2005) 165.
- [22] R. Contino, L. Da Rold, and A. Pomarol, Phys. Rev. D **75**, 055014 (2007).
- [23] G. 't Hooft, Nucl. Phys. B **72** (1974) 461; E. Witten, Nucl. Phys. B **160** (1979) 57.
- [24] See, for example, K. Lane, arXiv:hep-ph/0202255, and references therein.
- [25] R. Barbieri, A. Pomarol, R. Rattazzi, and A. Strumia, Nucl. Phys. B **703** (2004) 127.
- [26] M. E. Peskin and T. Takeuchi, Phys. Rev. Lett. **65**, 964 (1990); Phys. Rev. D **46**, 381 (1992).
- [27] K. Agashe, R. Contino, L. Da Rold, and A. Pomarol, Phys. Lett. B **641** (2006) 62.
- [28] G. Cacciapaglia, C. Csaki, G. Marandella *et al.*, Phys. Rev. **D75** (2007) 015003.
- [29] N. Arkani-Hamed, A. G. Cohen, and H. Georgi, Phys. Lett. **B513** (2001) 232–240; N. Arkani-Hamed, A. G. Cohen, T. Gregoire *et al.*, JHEP **0208** (2002) 020.
- [30] See, for example, G. L. Bayatian *et al.* [CMS Collaboration], J. Phys. G **G34** (2007) 995–1579.
- [31] K. Agashe, A. Belyaev, T. Krupovnickas, G. Perez, and J. Virzi, Phys. Rev. D **77** (2008) 015003.
- [32] R. Contino and G. Servant, JHEP **0806** (2008) 026; J. A. Aguilar-Saavedra, JHEP **0911** (2009) 030.
- [33] J. Mrazek A. Wulzer, Phys. Rev. **D81** (2010) 075006.
- [34] T. Han, H. E. Logan, B. McElrath *et al.*, Phys. Rev. **D67** (2003) 095004; M. Perelstein, M. E. Peskin, and A. Pierce, Phys. Rev. D **69** (2004) 075002.
- [35] G. Azuelos *et al.*, Eur. Phys. J. C **39S2** (2005) 13.
- [36] G. F. Giudice, C. Grojean, A. Pomarol, and R. Rattazzi, JHEP **0706** (2007) 045.
- [37] M. Dührssen, ATL-PHYS-2003-030.
- [38] J. A. Aguilar-Saavedra *et al.* [ECFA/DESY LC Physics Working Group], arXiv:hep-ph/0106315.
- [39] M. S. Chanowitz and M. K. Gaillard, Nucl. Phys. B **261** (1985) 379.
- [40] T. Kaluza, Sitzungsber. Preuss. Akad. Wiss. Berlin (Math. Phys.) **K1** (1921) 966.
- [41] M. B. Green, J. H. Schwarz, and E. Witten, *Superstring Theory*, (Cambridge Univ. Press, 1987).
- [42] N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, Phys. Lett. **B429** (1998) 263; Phys. Rev. D **59** (1999) 086004.
- [43] L. Randall and R. Sundrum, Phys. Rev. Lett. **83** (1999) 3370.
- [44] O. Klein, Z. Phys. **37** (1926) 895.
- [45] J. Polchinski, Phys. Rev. Lett. **75** (1995) 4724.
- [46] I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos, and G. R. Dvali, Phys. Lett. B **436** (1998) 257.
- [47] E. G. Adelberger, J. H. Gundlach, B. R. Heckel *et al.*, Prog. Part. Nucl. Phys. **62** (2009) 102–134.

- [48] G. F. Giudice, R. Rattazzi, and J. D. Wells, Nucl. Phys. B **544** (1999) 3.
- [49] M. Karagoz [CDF and D0 Collaboration], AIP Conf. Proc. **753** (2005) 400 [arXiv:hep-ex/0411067].
- [50] S. Cullen and M. Perelstein, Phys. Rev. Lett. **83** (1999) 268.
- [51] S. Hannestad, G. G. Raffelt, Phys. Rev. Lett. **88** (2002) 071301.
- [52] S. Dimopoulos and G. Landsberg, Phys. Rev. Lett. **87** (2001) 161602; S. B. Giddings and S. Thomas, Phys. Rev. D **65** (2002) 056010.
- [53] H. Davoudias, J. L. Hewett, and T. G. Rizzo, Phys. Lett. **B473** (2000) 43; A. Pomarol, Phys. Lett. **B486** (2000) 153.
- [54] For the effect of the dilaton see: J. Garriga and T. Tanaka, Phys. Rev. Lett. **84** (2000) 2778.
- [55] T. Gherghetta and A. Pomarol, Nucl. Phys. B **586** (2000) 141.
- [56] S. S. Gubser, Phys. Rev. D **63** (2001) 084017; N. Arkani-Hamed, M. Porrati, and L. Randall, JHEP **0108** (2001) 017; R. Rattazzi and A. Zaffaroni, JHEP **0104** (2001) 021.